Duong Van Tien TOPIC: Developing an integrated inventory model for energy optimization in imperfect production systems 2025

THE UNIVERSITY OF DANANG
UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF PROJECT MANAGEMENT

CAPSTONE PROJECT

MAJOR: INDUSTRIAL ENGINEERING AND MANAGEMENT

THESIS TOPIC

DEVELOPING AN INTEGRATED INVENTORY MODEL FOR ENERGY OPTIMIZATION IN IMPERFECT PRODUCTION SYSTEMS

Supervisor: **Dr. Nguyen Hong Nguyen**

Student: **Duong Van Tien**

Student id: 118200221

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Da Nang, June 2025

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ABSTRACT

The study proposes an Energy-Integrated EPQ Model (EEPQ_S) with four core components: stochastic demand following a Poisson distribution, imperfect production systems with defect rates, backordering policies, and energy consumption in both production and storage processes. This model optimizes production rates and cycle times to balance energy cost savings with maintained order fulfillment rates. To address the complexity of the objective function, the research employs optimization algorithms Grid Search implemented in Python. EEPQ_S demonstrates exceptional adaptability to fluctuating demand patterns, imperfect production conditions, and volatile energy prices, achieving superior economic and environmental efficiency compared to conventional models. Sensitivity analysis reveals that energy prices and electricity consumption significantly impact the model's performance. These findings hold substantial technical relevance while offering practical value for enterprises seeking to enhance operational efficiency and sustainability amid rising energy costs. The practical value is validated through the application of the study to Vinamilk Da Nang company.

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THESIS ASSIGNMENT

Student's Full Name: Duong Van Tien Student ID: 118200221

Class: 20QLCN2 Faculty: Project Management

Major: Industrial Engineering and Management

1. Thesis Topic: Developing an integrated inventory model for energy optimization in imperfect production systems

2. Thesis Category: ☐ Subject to an intellectual property agreement for the results

3. Initial Data and Information:

- The set of parameters applied to the study was referenced and adjusted from H.-N. Nguyen et al. and Sarkar et al.
- The set of parameters applied to the enterprise from the publicly available sources of Vinamilk.

4. Contents of report and calculations:

- Chapter 1: Introduction
- Chapter 2: Literature review
- Chapter 3: Mathematical model
- Chapter 4: Model development
- Chapter 5: Resolution approach and numerical analysis
- Chapter 6: Business applications and conclusion

5. Drawings and Charts (specify types and sizes of drawings): None

6. Supervisor's full name: PhD. Nguyen Hong Nguyen

7. Date of Thesis Assignment: ……/……/2025

8. Thesis Completion Date: ……/……/2025

Da Nang City, June 10, 2025

Head of Department Supervisor

Dr. Huynh Nhat To Dr. Nguyen Hong Nguyen

PREFACE

In the context of rising energy prices and increasing global pressure to reduce emissions, optimizing production activities to enhance economic efficiency in parallel with sustainable development has become both a theoretical and practical necessity. Motivated by this demand, I have chosen to undertake my thesis titled "Developing an Integrated Inventory Model for Energy Optimization in Imperfect Production Systems" at the Faculty of Project Management, University of Science and Technology – The University of Danang.

During the process of research and completion of this thesis, I have received valuable support, encouragement, and favorable conditions from many individuals and organizations. First and foremost, I would like to express my sincere gratitude to the Board of Directors and all lecturers of the University of Science and Technology – The University of Danang, especially those at the Faculty of Project Management, for providing a professional academic and research environment that has inspired and motivated me throughout my studies and the completion of this thesis. I would like to especially thank my supervisor, Dr. Nguyen Hong Nguyen, for his dedicated guidance, research orientation, and continuous support from the very beginning until the completion of the thesis. His insightful comments, rigorous yet sincere feedback, and professional discussions have been a significant source of motivation or me to constantly improve myself and the research work. I also extend my gratitude to all faculty members and supporting staff for their expertise, provision of materials, and encouragement throughout my academic journey. Most importantly, my sincere thanks go to Vinamilk Joint Stock Company for their cooperation, for providing practical data, and for sharing invaluable real-world experience, which greatly contributed to the practical application and relevance of this research. The enterprise's support serves as a vital bridge between theory and practice, enhancing the value and applicability of the thesis. The content of this thesis focuses on building and proposing an Energy-Integrated EPQ Model with four core components: stochastic demand following a Poisson distribution, imperfect production systems with defect rates, backordering policies, and energy consumption in both production and storage processes. This model aims to optimize production rates and cycle times, balancing energy cost savings with maintained order fulfillment rates. To address the complexity of the objective function, the thesis employs modern optimization algorithms Grid Search implemented in Python. demonstrates exceptional adaptability to fluctuating demand patterns, imperfect production conditions, and volatile energy prices, achieving superior economic and environmental efficiency compared to conventional models. Sensitivity analysis reveals that energy prices and electricity consumption significantly impact the model’s performance. These findings hold substantial technical relevance while offering practical value for enterprises seeking to enhance operational efficiency and sustainability amid rising energy costs.

Regarding structure, the thesis consists of six main chapters:

- Chapter 1: Introduction to the research context, motivation, objectives, scope, and significance of the topic.
- Chapter 2: Literature review, summarizing traditional EOQ/EPQ models and recent developments related to energy consumption, stochastic demand, imperfect production systems, and inventory policies.
- Chapter 3: Development of the mathematical model, including notation, assumptions, and practical analysis of the model components.
- Chapter 4: Model development, formulation of cost and revenue functions, and investigation of scenarios with and without shortages.
- Chapter 5: Resolution approach, application of optimization algorithms, numerical analysis, and sensitivity evaluation.
- Chapter 6: Business application at Vinamilk, conclusion, and future research directions.

Although I have made my best efforts, shortcomings are inevitable. I sincerely look forward to receiving feedback and guidance from esteemed teachers, scientists, and professionals to further improve this thesis.

Sincerely,

Da Nang City, June 10, 2025

Student

Duong Van Tien

DECLARATION

I am Duong Van Tien, student ID 118200221, class 20QLCN2, majoring in Industrial Engineering and Management, solemnly declare that my thesis entitled "Developing an Integrated Inventory Model for Energy Optimization in Imperfect" is my independent research work, conducted under the academic supervision of Dr. Nguyen Hong Nguyen. I affirm that all data, analyses, and results presented in this thesis are truthful, collected, and processed with utmost care, adhering to academic principles. The thesis does not contain any material copied from other sources without proper and full citation in accordance with academic regulations. I take full responsibility for the honesty, accuracy, and originality of the thesis content, and I further declare that this work has not been submitted for any other academic program or institution.

Da Nang City, June 10, 2025

Student

Duong Van Tien

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LIST OF SYMBOLS AND ABBREVIATIONS

SEC	Specific Energy Consumption
EOQ	Economic Order Quantity
EPQ	Economic Production Quantity
EEPQ	Energy-Integrated EPQ
EEPQ_S	Energy-Integrated EPQ with shortage
EPO	Energy Production Optimization
MPS	Master Production Scheduling
LM	Load Management
JIT	Just-In-Time
HC	Average Holding Cost Per Cycle ($/H)
PC	Energy Consumption Cost During Production Phase ($/H)
SC	Average Setup Cost Per Cycle ($/H)
IC	Average Energy Cost Incurred While The Machine Remains Powered In Periods without Production ($/H)
EC_{ware}	Average Warehouse Energy Cost Driven By Required Temperature And Storage Level ($/H))
DC	Average Defective-Item Cost Per Cycle ($/H)
SH	Average Shortage Cost Per Cycle In Shortage Case ($/H)
EPQ - Vinamilk	The Traditional Model
EEPQ - Vinamilk	The Energy-Integrated Improved Model

CHAPTER 1: INTRODUCTION

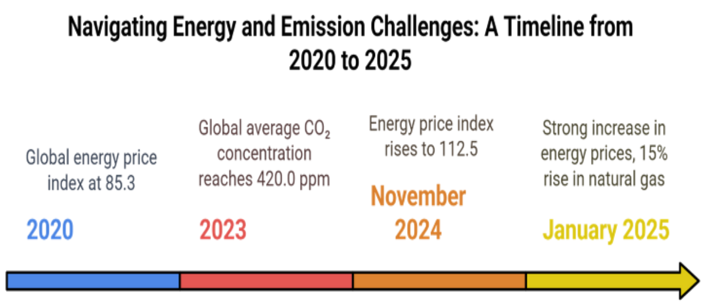
Chapter 1 lays the groundwork for the research by outlining the context and motivation, focusing on global energy price trends and emission challenges. It clarifies the research objectives, scope, and methodology, while highlighting the scientific and practical significance of the study. By analyzing pressing issues and defining a clear direction, this chapter establishes the theoretical and practical framework for the subsequent content.

1.1 Research Context and Motivation

1.1.1 Analysis of Energy Price Trends and Global Emission Challenges

According to the World Bank's *Commodity Markets Outlook* [1] report, global energy prices, encompassing electricity, natural gas, and fuels, recorded an increase of approximately 32% from 2020 to 2024. To corroborate, the International Monetary Fund's (IMF) global energy price index, known as PNRGINDEXM[2], rose from an average of 85.3 in 2020 to approximately 112.5 in November 2024, based on FRED data. The calculated rate of increase, [(112.5 - 85.3) / 85.3 × 100] ≈ 31.9%, closely aligns with the reported 32%, reinforcing the reliability of this trend. The World Bank's October 2024 [1] report further notes that commodity prices, including energy, remained over 30% higher than the five-year average prior to the COVID-19 pandemic, indicating that energy prices have sustained elevated levels throughout this period. Supplementary data from the IMF report highlights significant volatility in energy prices, with peaks in 2022 driven by supply disruptions.

The rise in global energy prices from 2020 to 2024 stems from multiple critical factors, creating a complex chain of impacts on energy markets. First, geopolitical tensions, notably the conflict in Ukraine beginning in 2022, severely disrupted the supply of natural gas and crude oil from Russia one of the world's largest suppliers. This shortage exerted significant pressure, particularly in Europe, causing energy prices to surge. Second, the economic recovery following the COVID-19 pandemic spurred a sharp increase in energy demand as industries and transportation resumed operations. However, uneven recovery across regions exacerbated global supply shortages. Finally, market volatility further intensified instability, with factors such as production constraints by OPEC+ and slow progress in transitioning to renewable energy sources leaving the world heavily reliant on fossil fuels. For instance, the World Bank report indicates that natural gas prices rose by 15% and crude oil by 8.1% in early 2025[3], reflecting the persistent price instability extending from 2024.



According to the *Greenhouse Gas Bulletin No. 20* by the World Meteorological Organization (WMO) [4], the global average CO_2 concentration reached a record high of 420.0 ppm in 2023, an increase of 2.3 ppm from 2022 (417.7 ppm). The report also indicates that over the past decade, the annual average increase in CO_2 has exceeded 2 ppm, reflecting the rapid accumulation of greenhouse gases. From 2004 to 2023, CO_2 concentrations rose by 11.4% (from 377.1 ppm to 420.0 ppm), underscoring an alarming rate of increase. The industrial sector, contributing over 30% of global emissions, faces a target of reducing CO_2 emissions by 24.2%, as outlined in Ritchie's (2020) [5] study. However, WMO data suggests that global efforts remain insufficient to reverse the rising emissions trend, particularly amidst high energy prices that increase the cost of transitioning to clean technologies. The increase in CO_2 emissions from 2020 to 2024 can be attributed to three primary factors, reflecting the complexity of global environmental challenges. First, the post-COVID-19 economic recovery significantly increased energy demand, particularly in manufacturing and transportation sectors. The widespread use of natural gas and crude oil high CO_2 emitting fuels substantially contributed to the rise in atmospheric greenhouse gas concentrations. Second, the transition to clean energy sources has faced constraints, despite policies such as carbon taxes and investments in renewables. This transition has progressed more slowly than anticipated, leaving many countries reliant on fossil fuels to meet growing energy demands. Finally, natural factors have played a significant role. The WMO report notes that the El Niño phenomenon and wildfires in late 2023 increased CO_2 emissions while reducing the efficacy of natural carbon sinks such as forests and oceans [4]. Specifically, just under 50% of emitted CO_2 remains in the atmosphere, with over 25% absorbed by oceans and nearly 30% by terrestrial ecosystems, but annual fluctuations due to natural phenomena have further complicated this process. This situation calls for advanced solutions, like energy-integrated inventory models, to balance economic efficiency and environmental responsibility in volatile energy markets and rising sustainability demands.

1.1.2 Research Background

Amid rising energy costs and the need for sustainable development, energy-efficient production planning is a top priority for manufacturers. Decision support models are crucial for optimizing production, reducing costs, and meeting environmental goals like lowering greenhouse gas emissions. However, traditional EOQ and EPQ models often fail to incorporate variable energy factors, limiting their ability to reflect real-world market dynamics and modern production systems. Advanced models like EPO and EEPQ address this by optimizing both production and energy costs. Biel and Glock (2016) [6] offer a comprehensive review, categorizing these models and highlighting the potential of energy-integrated approaches in production management.

Biel and Glock (2016) [6] classify energy-integrated production planning models into five groups: master production scheduling (MPS), lot-sizing, machine scheduling, load management (LM), and load tracking (LT). The EPQ model excels due to its ability to simplify short-term production decisions while integrating energy costs and handling stochastic demand. Compared to MPS, which balances medium-term capacity but struggles with volatile demand (e.g., Poisson distribution) due to stable demand assumptions and limited focus on per-cycle quantities, EPQ determines economic production quantities, balancing setup and holding costs for rapid adaptation to uncertainties. Machine scheduling optimizes task sequencing to reduce peak-hour electricity use but is complex, requiring detailed machine/task data, and is unnecessary when the goal is optimal production quantities. LM and LT focus on adjusting electricity loads to cut costs under contracts (e.g., flattening load profiles), but they neglect production and inventory costs.

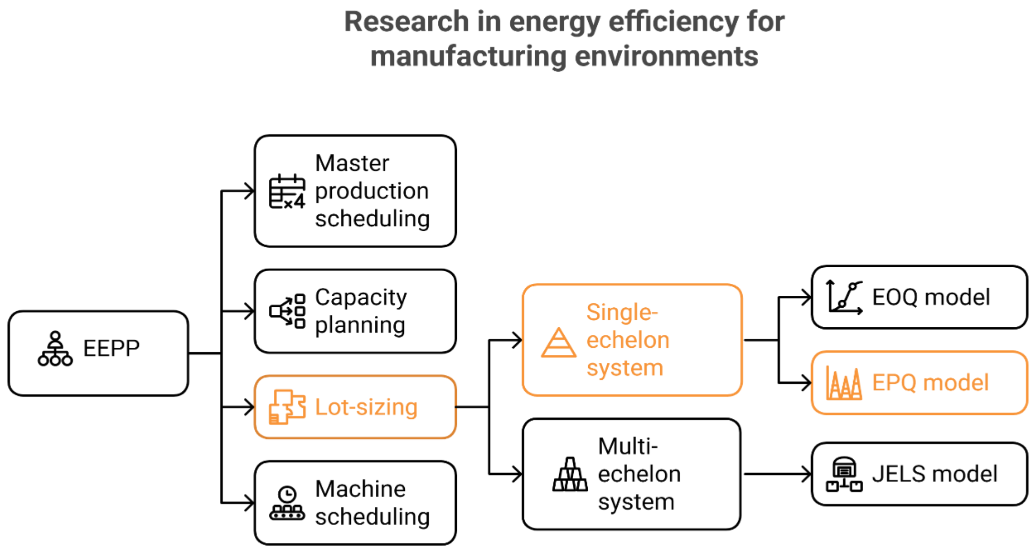


Figure 1.1 Research in enegy efficiency for manufacturing environments

Traditional EOQ and EPQ models, while foundational for inventory and production management, have significant limitations in energy-efficient production planning. They often ignore variable energy costs, focusing only on ordering, holding, and setup costs, failing to account for fluctuating electricity prices under time-of-use (TOU) tariffs or peak demand fees. For instance, a factory operating during peak hours may face double the electricity costs compared to off-peak, but EOQ/EPQ models cannot adjust schedules. They also assume stable demand, ignoring peak/off-peak hours or contractual constraints, leading to overproduction or shortages that increase energy use. Additionally, they overlook flexible energy policies like interruptible load contracts or TOU pricing, limiting their applicability in energy-sensitive, volatile markets (Biel & Glock, 2016) [6]. In contrast, the EPO (EEPQ) model integrates energy costs, flexible pricing, and load management, optimizing operations. It reduces total costs, enhances flexibility, and aids CO2 emissions reduction, enabling businesses to adapt to fluctuating electricity prices and environmental mandates, shaping sustainable production (Biel & Glock, 2016) [6].

1.2 Research Objectives, Scope and Methodology

1.2.1 Research Objectives

This study focuses on developing an advanced inventory model, termed EEPQ_S. The primary objective of this model is to optimize energy consumption in imperfect production systems when facing stochastic demand. EEPQ_S is designed to overcome the limitations of traditional models such as the EOQ and EPQ, which often fail to adequately consider real-world factors such as demand variability, defective product rates, energy costs, and flexible inventory management policies. The study integrates four critical aspects:

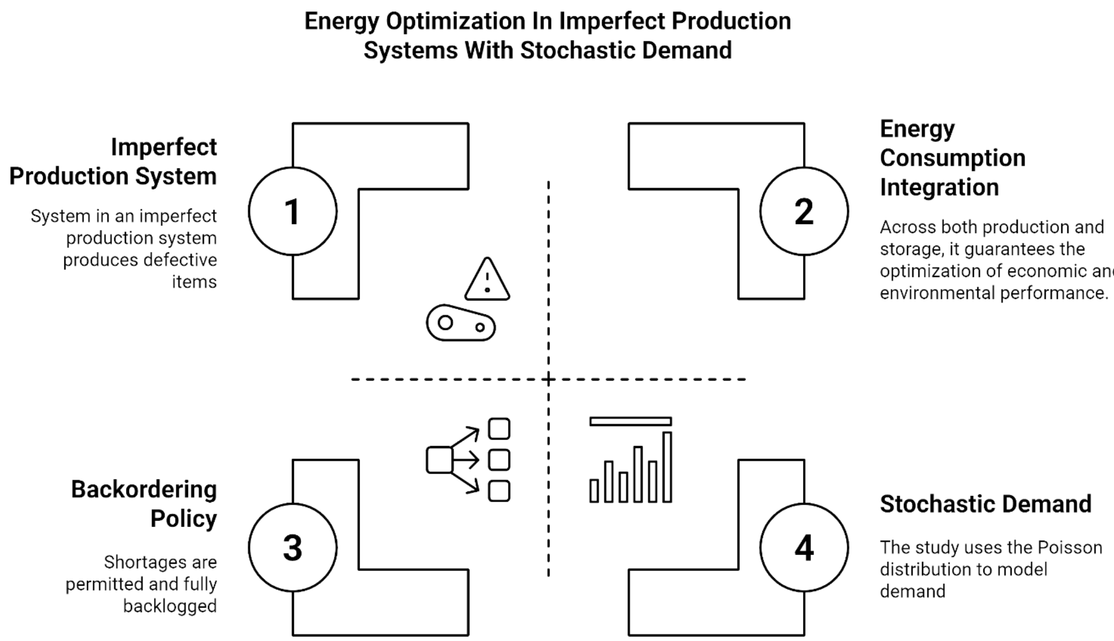


Figure 1.2 Energy optimization in imperfect production systerms with stochastic demand.

- **Stochastic demand:** Utilizes a Poisson distribution to model market demand, reflecting the uncertainty and unpredictability of real-world demand.
- **Imperfect production process:** Accounts for the rate of defective products during production, representing the practical production conditions faced by most enterprises.
- **Backordering policy:** Permits the recording of unmet demand during inventory shortages and fulfilling it after restocking, enhancing the system's flexibility and responsiveness.
- **Energy consumption:** Incorporated into the model for both production and storage phases, calculated based on specific energy consumption and factors such as warehouse temperature, inventory occupancy levels, and storage duration.

1.2.2 The specific objectives of the study include

To address the challenges in inventory management and energy optimization, this study proposes an integrated model to enhance efficiency in imperfect production systems. The specific objectives of the study include:

- Optimizing production rate and production cycle: Determining optimal values for production rate and production cycle to achieve a balance between minimizing energy costs, production costs, storage costs, and backordering costs, while ensuring a high level of demand fulfillment.
- Evaluating the adaptability of the EEPQ_S model: Assessing the model's flexibility in addressing changing real-world conditions, including demand variability, varying defective product rates, and fluctuations in energy prices. Additionally, comparing the performance of EEPQ_S with traditional inventory models using metrics such as total cost, energy consumption, and order fulfillment rate.
- Analyzing sensitivity to key parameters: Conducting sensitivity analysis to evaluate the impact of factors such as energy prices, electricity consumption levels, defective product rates, and backordering costs on the model's overall performance. Identifying the sensitivity of decision variables to changes in these parameters, thereby providing detailed insights to support decision-making under uncertainty.

These objectives not only address the limitations of traditional models but also aim to develop a comprehensive, sustainable inventory management solution, particularly in the context of rising energy prices and increasing pressure to reduce CO_2 emissions.

1.2.3 Scope and Methodology

The research methodology is carefully designed to meet the stated objectives. The EEPQ_S model extends the traditional EPQ model, incorporating stochastic demand (Poisson distribution), imperfect production, backordering, and energy consumption. Mathematical equations model relationships between production rate, cycle, energy costs, and backordering costs. Assumptions include a single-item system, Poisson-distributed demand with a fixed mean, constant defective rate, and energy use proportional to production rate and storage conditions. The objective is to maximize average profit or minimize total costs (setup, holding, energy, defective products, and backordering), with production rate and cycle as key variables. Due to the nonlinear, stochastic objective function, advanced optimization algorithms are used, coded in Python with NumPy, SciPy, and Matplotlib. Input parameters (setup costs, holding costs, defective rates, energy parameters) are sourced from prior studies and industry data. EEPQ_S is tested in realistic scenarios with varying demand, energy prices, and defective rates, compared to EPQ on total costs, energy use, and demand fulfillment. Sensitivity analysis examines parameter impacts. In evaluation, EEPQ_S is benchmarked against traditional models using metrics like average profit, energy consumption, and order fulfillment rates. Simulations assess feasibility under real-world conditions, such as peak demand or energy price volatility.

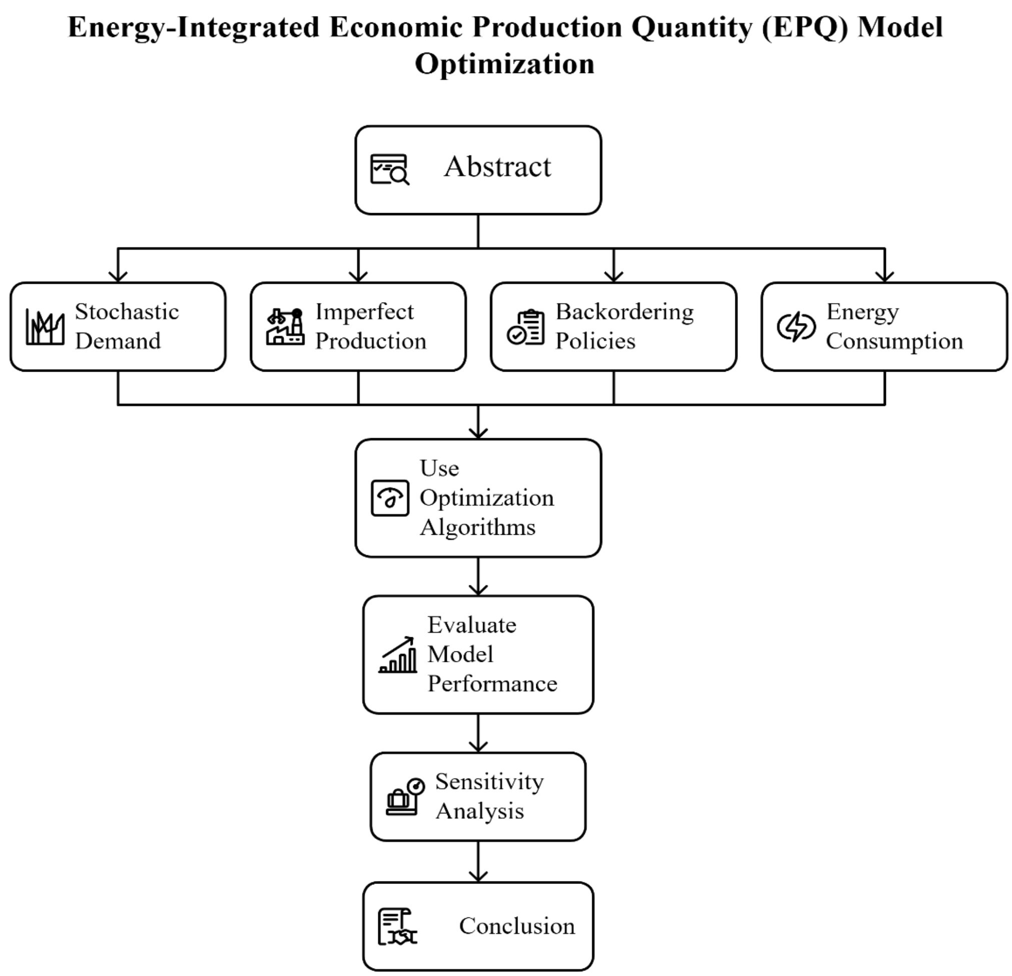


Figure 1.3 Energy – Integrated EPQ optimization

The study concludes by synthesizing key findings and providing recommendations for businesses on implementing EEPQ_S, while also suggesting future research directions, such as extending the model to multi-item systems or integrating renewable energy sources.

This methodology ensures a comprehensive approach, bridging theory and practice, and establishes a robust foundation for the deployment of the EEPQ_S model.

1.3 Scientific and Practical Significance

The study enhances traditional inventory models like EOQ and EPQ by addressing two key limitations: fixed demand assumptions and the absence of energy and emissions factors. Firstly, the EEPQ_S model incorporates stochastic demand via a Poisson distribution, capturing real-world market variability often ignored by traditional models due to their reliance on stable, predictable demand. This is supported by Widyadana and Wee (2012)[7], who emphasized the need to extend EPQ models to account for stochastic demand to enhance practical applicability.

Secondly, The study addresses the critical limitation of traditional models in neglecting energy and emissions factors. Per Gutowski et al. (2006) [8], modern manufacturing involves optimized processing steps, especially in large-scale production, often automated for efficiency and precision. Some processes integrate all steps into one machine, simplifying operations and workflows. For example, a modern milling machine not only cuts metal but also handles workpiece management, lubrication, chip removal, automatic tool changing, and tool break detection. These auxiliary functions significantly contribute to energy consumption. In an automotive machining line (Figure 1.4), actual machining consumes only 14.8% of total energy, with the rest used for auxiliary tasks like cooling and control systems. At lower production rates, the machining energy proportion shrinks further, highlighting the dominance of auxiliary functions. This pattern extends beyond metal machining to various manufacturing processes.

The content further indicates that energy consumption in manufacturing processes can be divided into two primary components:

- Start-up and standby energy (Po): This is the energy required to initiate equipment and maintain it in a “ready” state, even when no material is being processed. For instance, in a cutting machine, Po encompasses the energy for coolant pumps, hydraulic pumps, computer control panels, and other devices in an idle state. In a thermal process, Po represents the energy needed to maintain a furnace at a stable operating temperature.
- Energy proportional to the material processing rate (kv): This component depends on the material processing rate (v, measured in cm^3/second) and a characteristic constant

(k, measured in kJ/cm³), which is related to the physical properties of the process. For example, in cutting operations, k depends on the material's hardness and the mechanics of the cutting process; in thermal processes, k is associated with the heat required to raise the temperature of a unit of product.

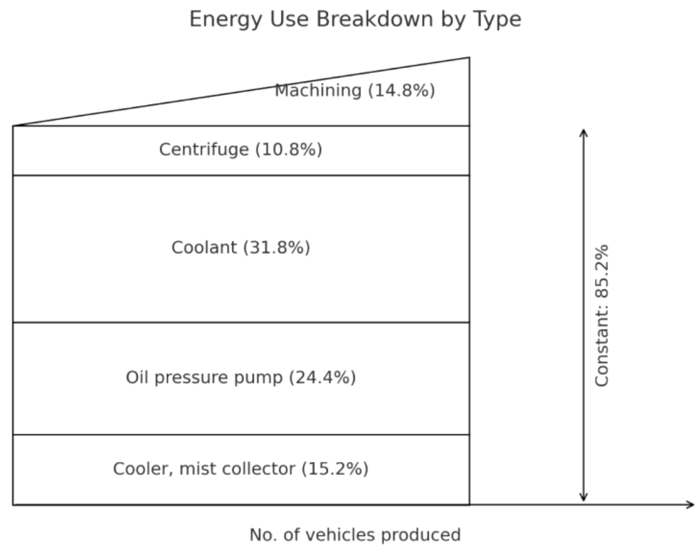


Figure 1.4 Energy used as a function of production rate for an automobile production machining line per [9]

This relationship is modeled in the equation: $$P = P_0 + kv \tag{1}$$

Based on this equation, Equation 1 is derived to calculate the specific electrical exergy (available energy per unit of processed material, Belect, measured in kJ/cm³):

$$B_{elect} = \frac{P_0}{v} + k \tag{2}$$

Where Po represents the exergy associated with the idle state, and k reflects the energy required for the actual processing. This equation demonstrates that energy consumption is not solely dependent on the processed material but is also significantly influenced by the operational requirements of the equipment.

Empirical evidence for this relationship is presented in the illustrative figure 1.5 . Total energy consumption is divided into:

- Variable energy (65.8%): This is the energy that varies with the equipment's load, primarily associated with the actual machining process. Machining (65.8%): This portion of energy directly supports the machining process, specifically metal cutting through plastic deformation. However, as noted in the content, this energy accounts for only 14.8% of the total energy consumption of the entire system, indicating that even within the variable energy component, machining is not the sole contributor. Other components within the variable energy may include load-related activities, though these are not detailed in the diagram.

- Constant energy (34.2%): This is the energy required to maintain the equipment in a ready state or to operate auxiliary functions, independent of the load.

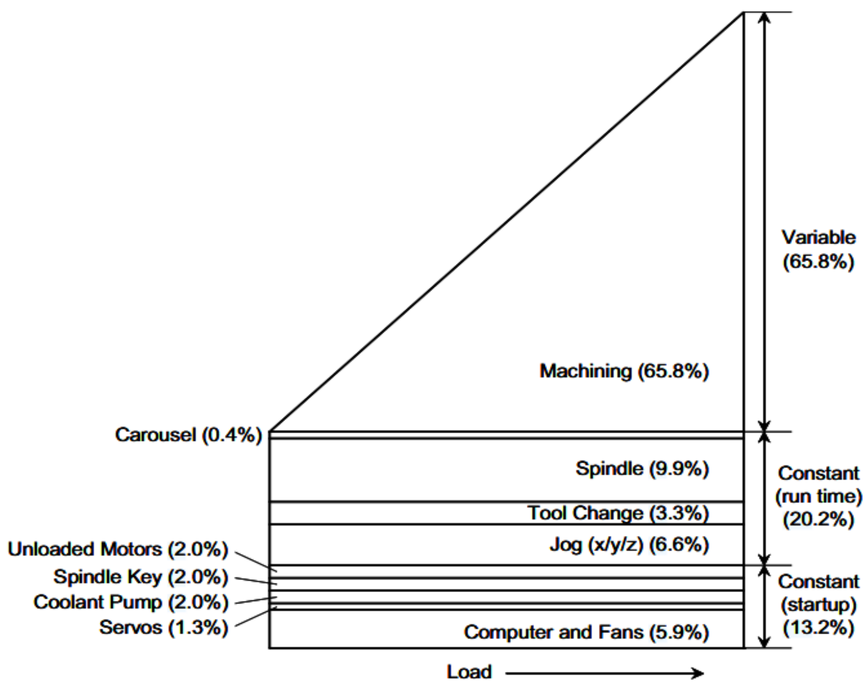


Figure 1.5 Energy used as a function of material removal rate for a 3-axis CNC milling machine [10]

The diagram shows that in a machining production line, energy consumption is heavily influenced by auxiliary functions and idle states, not just the primary machining process. This aligns with the energy model (Equations 1 and 2), where idle power (Po) and auxiliary factors are critical. These findings highlight the need for improved machine design and processes to reduce unnecessary energy use, promoting sustainable production. Machining data confirms energy consumption aligns with Equations 1 and 2. The EEPQ_S model, integrating Specific Energy Consumption (SEC) into production and inventory, addresses both economic and environmental impacts. Marchi et al. (2019) [11] show that energy-efficient strategies like “SEC synchronization” optimize costs under fluctuating demand, underscoring the importance of energy considerations in inventory management.

The EEPQ_S model also contributes to theoretical advancements by addressing imperfect production systems, where a proportion of products are defective, and backordering policies, which have not been thoroughly explored in traditional models. By optimizing production time and rates, EEPQ_S enables businesses to reduce operating costs, improve efficiency, and maintain competitiveness in volatile markets. For instance, adjusting production rates to reduce SEC, as highlighted by Glock & Grosse (2021)[12], can lead to significant energy savings in energy-intensive industries such as metallurgy or paper production. Moreover, the study contributes to sustainability goals by minimizing CO_2 emissions through optimized energy consumption. Companies adopting this model can not only reduce costs but also enhance their brand image through a commitment to sustainable development.

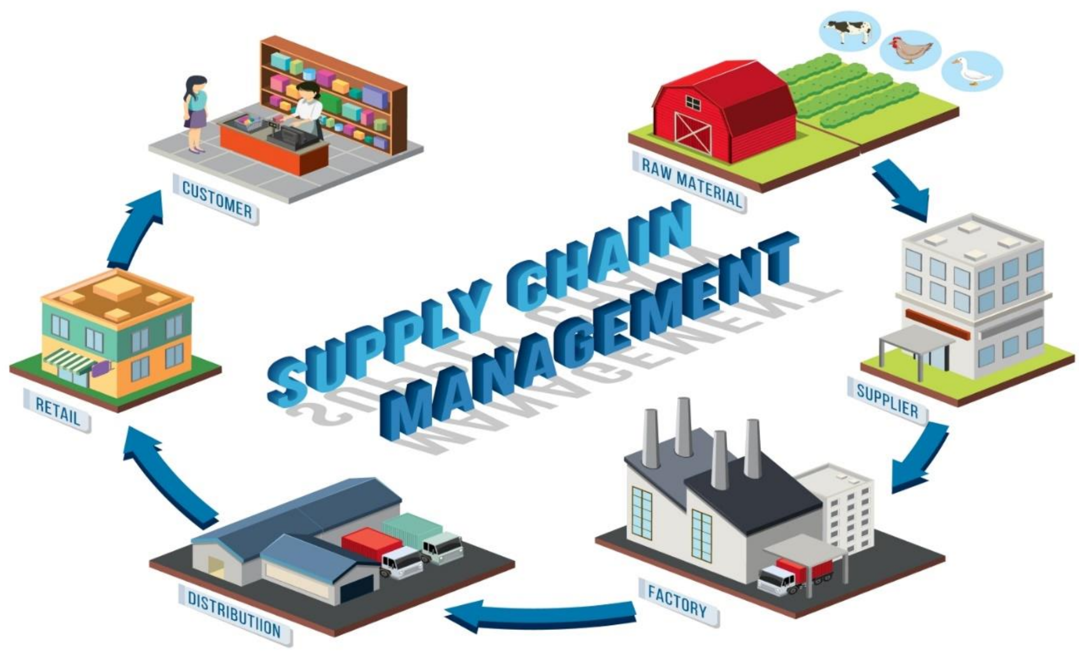


Figure 1.6 Supply chain managemaent

This research bridges theory and practice by integrating factors such as energy consumption, stochastic demand, and imperfect production systems into a comprehensive optimization model. The results not only enrich the theoretical foundation but also provide practical tools for businesses to improve operational performance. This study is particularly significant, contributing to the global advancement of sustainable and efficient industrial practices.

CHAPTER 2: LITERATURE REVIEW

Chapter 2 offers a comprehensive review of the literature, focusing on the foundational concepts of the EOQ and EPQ models. It explores traditional modeling systems and delves into recent advancements, including the integration of sustainable energy considerations (SEC) into the EPQ model, Poisson-distributed stochastic demand models, and backorder policies in imperfect production systems. By identifying research gaps and proposing future research directions, this chapter establishes a critical theoretical foundation for addressing contemporary challenges in inventory and production management.

2.1 Traditional EOQ/EPQ Modeling System

2.1.1 Economic Order Quantity (EOQ) Model

The EOQ model, introduced by Ford Whitman Harris in 1913[13], is one of the oldest and most widely used inventory management models. This model determines the optimal order quantity that a company should purchase from a supplier to minimize total inventory costs, which include ordering costs and holding costs. EOQ is particularly suitable for businesses that purchase goods rather than produce them.

The standard EOQ formula is:

$$EOQ = \sqrt{\frac{2DS}{H}} \tag{3}$$

Where:

- D: Annual demand (units per year), S: Ordering cost per order (VND/order),
- H: Holding cost per unit per year (VND/unit/year).

The EOQ model is based on the following assumptions:

- Demand is fixed and known in advance, remaining constant over time.
- Stockouts are not permitted.
- The entire order quantity is delivered instantaneously (infinite replenishment rate).
- No quantity discounts are available for larger orders.
- Inventory planning is considered over an infinite time horizon.
- Lead time is fixed and known.

According to Operations Management, Thirteenth Edition by William J. Stevenson[14], the figure titled The Inventory Cycle: Profile of Inventory Level Over Time illustrates the inventory cycles in the EOQ model:

- A cycle begins when an order of Q units is received, which is consumed at a constant rate.
- When the inventory level is sufficient to meet demand during the lead time, a new order (Q units) is placed.
- Due to the constant consumption rate and fixed lead time, the new order arrives precisely when the inventory level reaches zero, avoiding both excess inventory and shortages.

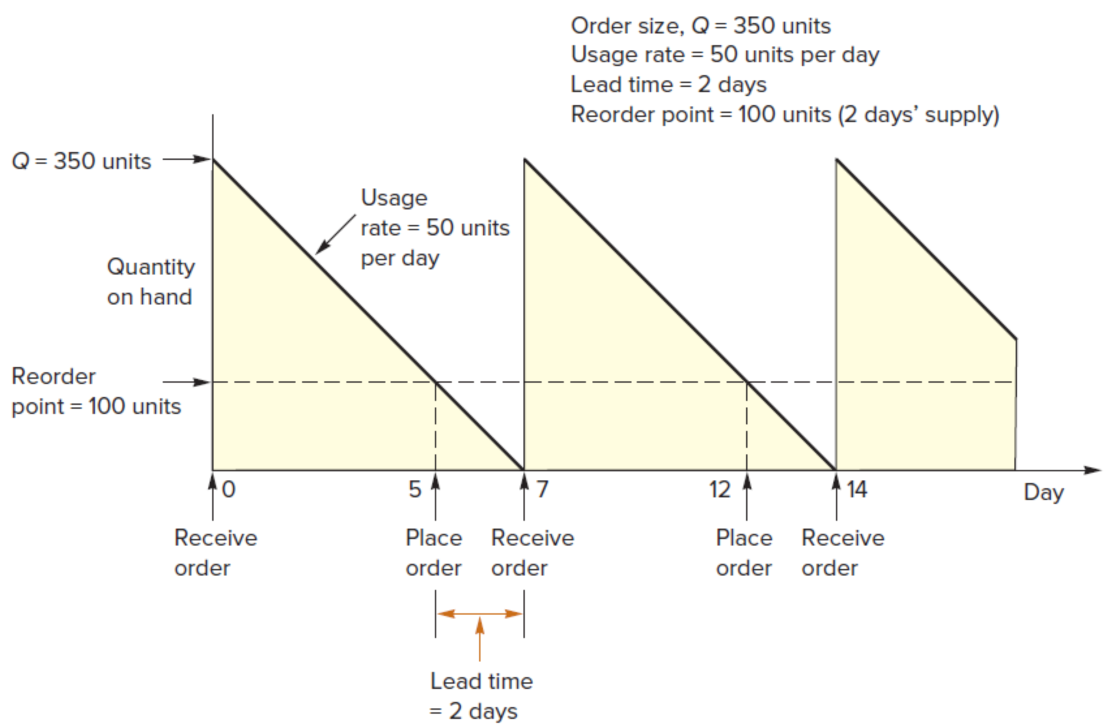


Figure 2.1: The inventory cycle: profile of inventory level over time

- Diagram: Starts at 350 units (upon receiving the order), decreasing steadily at a rate of 50 units/day. When inventory drops to 100 units (day 5), a new order is placed. After 2 days (lead time), inventory reaches 0 (day 7), a new order (350 units) is received, and the cycle repeats.
- Cycle: One cycle lasts 7(days (from day 0 to day 7, then repeats from day 7 to day 14). The reorder point and lead time ensure no shortages or excess inventory.

- Analyze the Inventory Cycle: Based on Q = 350 units and a usage rate of 50 units/day, we calculate the duration of one cycle (the time to consume 350 units):

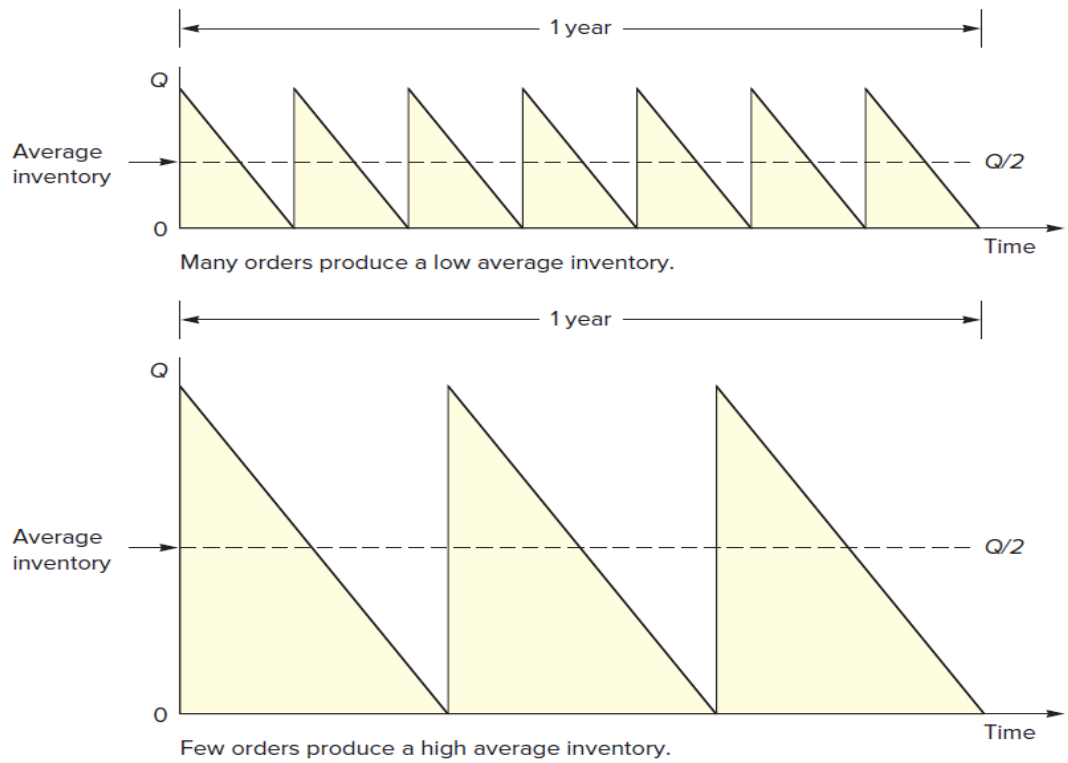


Figure 2.2 Average inventory level and number of orders per year are inversely related: As one increases, the other decreases

- Illustration of the relationship between order quantity, average inventory level, and number of orders per year: A smaller order quantity (compared to Q = 350 in Figure 2.2) results in more cycles per year (more frequent orders). The average inventory level is indicated by a horizontal line, calculated as Q/2.
  Note: "The more orders placed, the lower the average inventory level" – meaning that frequent ordering (smaller order sizes) reduces average inventory, but increases the number of orders.
- Advantages and Limitations: The EOQ model provides a simple and effective approach to inventory management by balancing ordering costs and holding costs. However, it has significant limitations when applied in practice. The assumption of fixed demand is unsuitable for industries where demand fluctuates due to market changes, seasonal variations, or unexpected events, as highlighted in the study by Ritchie and Roser

(2020)[5]. Furthermore, EOQ does not incorporate energy and emissions factors, leading to inaccurate assessments of actual costs, particularly in the context of rising energy prices and increasing pressure to reduce CO_2 emissions. For example, energy costs related to transportation or storage of goods are not considered, reducing the model's practicality in modern systems.

In the context of your research, EOQ serves as a starting point for understanding fundamental inventory management principles. However, to address limitations such as unstable demand and energy costs, the model requires extension. For instance, integrating energy consumption into the cost function could account for transportation or cold storage costs. Additionally, to handle stochastic demand, the model can be adjusted to consider demand following a Poisson distribution, as proposed in your study, better reflecting market uncertainty.

2.1.2 Economic Production Quantity (EPQ) Model

The EPQ model, developed by Taft in 1918, is an extension of the EOQ model, specifically designed for production environments where goods are manufactured internally rather than purchased externally. EPQ accounts for a finite production rate, meaning inventory accumulates gradually during the production phase. The standard EPQ formula is:

$$EPQ = \sqrt{\frac{2DS}{H\left(1-\frac{D}{P}\right)}} \tag{4}$$

Where:

- D: Annual demand (units per year),
- S: Ordering cost per order (VND/order),
- H: Holding cost per unit per year (VND/unit/year).
- P: Production rate (units per year, with $P > D$).
- $\frac{D}{P} = d$: Demand rate(units/day)

The EPQ model is based on the following assumptions:

- Demand is fixed and known.
- The production rate is fixed and greater than the demand rate.
- Stockouts are not permitted.

- Inventory planning is considered over an infinite time horizon.
- Setup costs are incurred for each production run.
- Holding costs apply to the average inventory level, including both produced inventory and held inventory.

In the EPQ model, the inventory never reaches the production quantity Q (here, EPQ = 698 units) because products are consumed continuously during production. The maximum inventory level I_{max} occurs at the end of the production phase and is calculated using the formula:

$$I_{max} = Q\left(1 - \frac{D}{P}\right) \tag{5}$$

The total cost is calculated using the following formula:

$$TC_{min} = Carrying\ Cost + Setup\ Cost = \left(\frac{I_{max}}{2}\right)H + \left(\frac{D}{Q}\right)S \tag{6}$$

Carrying cost (average inventory holding cost):

$$Carrying\ Cost = \left(\frac{I_{max}}{2}\right)H \tag{7}$$

Setup cost: $Setup\ Cost = \left(\frac{D}{Q}\right)S$ (8)

The inventory graph takes the form of a sawtooth pattern because inventory gradually increases during the production phase, then gradually decreases during the non-production phase, and the cycle repeats.

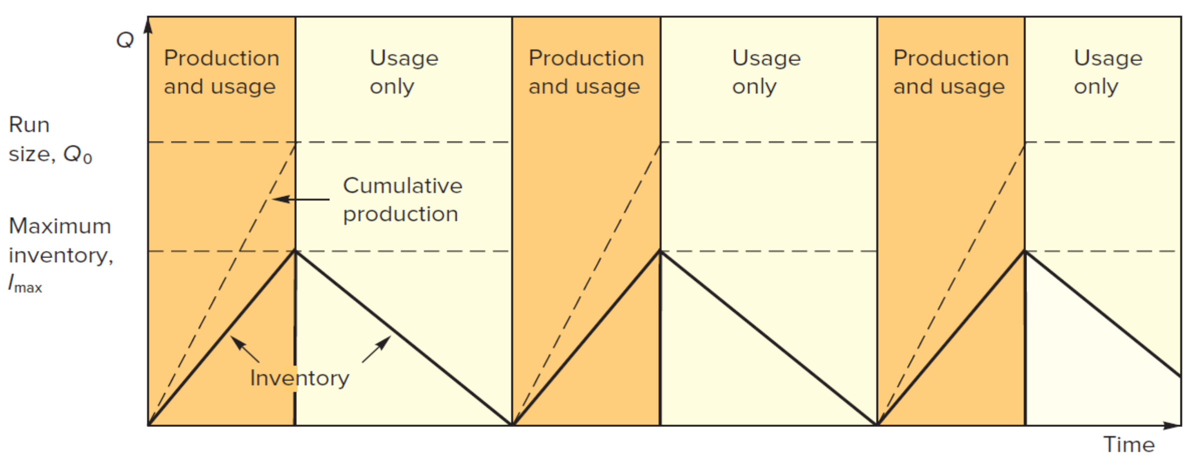


Figure 2.3 EPQ with incremental inventory buildup

In a production cycle for a specific item, inventory grows during the production phase at a rate equal to the production rate minus the usage rate. For example, if production is 20 units/day and usage is 5 units/day, inventory increases by 15 units/day. It rises until production stops, reaching its maximum level, then decreases at the constant usage rate. When inventory is depleted, production restarts, and the cycle repeats. Since the product is made in-house, there's no ordering cost, but each production run has a setup cost for equipment preparation (e.g., cleaning or adjusting tools). The setup cost, similar to ordering cost, is independent of lot size and calculated as (D/Q)S, where D/Q is the number of production runs per year and S is the setup cost per run. Larger lot sizes reduce the number of runs, lowering annual setup costs. The total cost is given by formula (6), where I_{max} is the maximum inventory level and H is the holding cost per unit per year. Unlike the EOQ model, continuous usage during production means inventory never reaches the full lot size Q, staying lower. The EPQ model is useful for companies that manufacture goods, as it takes into account the time required for production and the dynamics of inventory accumulation. During the production phase, inventory increases at a rate of $P - D$, and during the non-production phase, it decreases at a rate of D. However, similar to EOQ, the EPQ model assumes constant demand, which does not reflect reality in volatile markets, as noted in the study by Marchi et al. (2019)[11]. Furthermore, the traditional model does not incorporate energy and emission factors, even though energy costs can represent a significant portion of total production cost. According to Gutowski et al. (2006)[8], SEC depends on the production rate, and is given by the (2). Where B_{elect} is the SEC, P_0 is the idle power, v is the production rate, and k is a process-specific constant. This indicates that a higher production rate can reduce SEC, but it must be optimized to balance against other costs.

2.2 Recent developments

2.2.1 Integration of SEC into the EPQ model

In the context of rising global energy prices and increasing pressure to reduce greenhouse gas emissions, optimizing energy consumption in production and inventory management has become a top priority. Traditional models such as the EOQ and the EPQ often overlook the energy factor, leading to production decisions that do not fully reflect actual costs or environmental impacts. To address this limitation, researchers have begun integrating Specific SEC into inventory management models to balance economic efficiency with environmental sustainability.

The study by Zanoni et al. (2014)[15] represents a significant advancement in integrating SEC into the EPQ model, focusing on a two-stage production system with adjustable production rates. This system includes two consecutive machines (M1 and M2), where output from M1 is transferred to M2 for further processing. Zanoni and colleagues[15] extended the traditional EPQ model by adding energy costs to the total cost function, with SEC as a core component. The energy-related production cost of each machine is calculated based on SEC and is expressed as follows:

Machine	**Energy cost formula**
M_1	$PC_{M_1} = \frac{W_1}{P_1} + k_1 \times d \times e \quad (9)$
M_2	$PC_{M_2} = \frac{W_2}{P_2} + k_2 \times d \times e \quad (10)$

Explanation of parameters:

- $W_1;\ W_2$: Rated power of machines M_1 and M_2 (units: kW)
- $P_1;\ P_2$: Production rate of machines M_1 and M_2 (units: units/hour)
- $k_1;\ k_2$: Fixed energy consumption coefficients of each machine (units: kWh/product)
- d: Product demand rate (units: products/time); e: Energy price (units: €/kWh)

In the above formulas: The components $W_i;\ P_i$ represent the SEC, i.e., the energy consumed per unit of product due to the machine's production activity. As the production rate P_i increases, SEC decreases because the fixed energy consumption of the machine is distributed over a larger number of units within the same time frame. The coefficient k_i represents fixed energy consumption that does not depend on the production rate, such as the energy required to start the machine or to maintain operation in idle mode. The model proposed by Zanoni et al. accounts not only for energy costs but also incorporates other traditional EPQ cost components, including: Setup cost. holding cost, production cost(including energy costs calculated based on SEC).

By including SEC in the total cost function, the model enables simultaneous optimization of two key factors:

1. Production rate (P_i): A higher production rate helps reduce SEC, but it may increase holding costs if products are manufactured faster than they are consumed.
2. Lot size (Q): The optimal lot size must balance setup costs (which decrease with fewer production runs) and holding costs (which increase with larger lot sizes).

The optimization method used in the study involves mathematical analysis (based on derivatives) and simulation to determine the optimal values for P_i and Q that minimize total cost. In addition, Zanoni et al. conducted sensitivity analyses to assess the impact of various factors such as energy price (e), machine power rating (W_i), and production rate (P_i) on total cost and energy efficiency. Notable findings include:

- As energy prices rise, optimizing production speed becomes more critical, since energy costs represent a larger portion of total cost.
- Energy-saving policies, such as shutting down machines during idle times, can reduce energy-related costs by up to 54%. For example, if a machine consumes 10 kW in idle mode, shutting it off during non-production periods can yield significant energy savings.
- Increasing the production rate from 500 units/hour to 750 units/hour can reduce SEC from 0.02 kWh/unit to 0.013 kWh/unit, though this must be weighed against the potentially higher holding costs.

Following the study by Zanoni et al. (2014)[15] on the integration of Specific Energy Consumption (SEC) into the EPQ model for two-stage production, Marchi and Zanoni (2022)[16] expanded the scope of energy analysis from individual production processes to the entire supply chain, with a focus on the cold supply chain in the food and beverage industry. The cold supply chain plays a crucial role in preserving perishable food such as meat, fish, dairy products, and vegetables, but at the same time consumes a large amount of energy to maintain the required temperature. According to the International Institute of Refrigeration (2025)[17], the cold chain accounts for approximately 15% of the total energy used in the global food industry and contributes significantly to CO_2 emissions. Therefore, optimizing energy usage in the cold chain not only reduces costs but also promotes environmental sustainability.

Marchi and Zanoni (2022)[16] proposed a comprehensive approach to assess energy performance in the supply chain in order to identify inefficiencies and implement improvement measures. This study focuses on mapping energy flow across stages from farm to fork, including production, storage, transportation, and distribution, with the aim of reducing SEC and improving overall sustainability. Other studies have also contributed to integrating energy into inventory management. Nezami and Heydar (2011)[18] analyzed the impact of fluctuating energy prices on the energy-aware EPQ model, emphasizing that changes in energy prices can significantly influence optimal production decisions. Meanwhile, Marchi, Zanoni, and Jaber (2019)[11] and Marchi and Zanoni (2022)[16] extended this research by considering energy consumption in storage, including factors such as warehouse volume and ambient temperature, providing a more comprehensive view of energy costs in the supply chain. The empirical study by H.-N. Nguyen et al. (2024) [19] supports these findings, showing that a 40% increase in the energy consumption parameter leads to a 25% increase in total cost, highlighting the necessity of integrating detailed energy data.

The study by Zanoni et al. (2014)[15] laid the foundation for integrating Specific Energy Consumption (SEC) into the Economic Production Quantity (EPQ) model in a two-stage production system, where SEC is defined as the energy consumed per unit of product (kWh/product) and is included in the energy cost using the formula $PC_{M_1} = \frac{W_1}{P_1} + k_1 \times d \times e$ for machine M1, and similarly for machine M2. This model allows for the optimization of production rate and lot size, reducing energy costs by up to 54% through policies such as shutting down machines during idle periods, while also lowering CO_2 emissions in the context of industry accounting for 30% of global emissions according to Our World in Data (2024). Building on this research, Marchi and Zanoni (2022)[16] expanded energy analysis to the entire cold supply chain in the food and beverage industry, mapping energy flow from “farm to fork” and calculating SEC for production, storage, and transportation stages using a Weibull-power law model to balance energy consumption and product quality. From a management perspective, both studies promote supply chain collaboration, from optimizing factory production speed to co-investing in high-efficiency refrigeration technologies, aligning with industrial symbiosis for resource optimization. Future research includes integrating defective products and stochastic demand into the EPQ model, extending the cold chain approach to other industries, developing automated energy data collection tools for small businesses, and incorporating socio-economic factors, such as the impact of reduced energy costs on retail prices.

2.2.2 Poisson distributed stochastic demand model

The classical EOQ model, a cornerstone of inventory management, assumes constant and predictable demand. However, this does not align with market realities where demand fluctuates due to consumer trends, economic volatility, or unexpected events. Businesses using traditional EOQ risk stockouts, leading to lost sales, or excess inventory, increasing holding costs and spoilage risks. To address this, pioneering research has focused on modeling stochastic demand to better capture market uncertainty. A key breakthrough came from Patrick Alfred Pierce Moran (1959), who treated demand as a random variable rather than a fixed constant. Moran's work laid the foundation for stochastic inventory theory, offering a new approach to managing inventory in uncertain environments, enabling businesses to better handle fluctuations, reduce risks, and improve economic efficiency.

Building upon Moran's work, George Hadley and Thomson M. Whitin (1963), in their book *Analysis of Inventory Systems*[20], developed more detailed inventory models that used probability distributions to describe demand variability. These authors introduced advanced mathematical methods, including the application of Poisson and exponential distributions, to analyze demand behavior under various scenarios. Their work not only reinforced stochastic inventory theory but also provided practical tools for managers to make decisions regarding order quantities and replenishment timing, particularly in industries with unstable demand.

In a more modern context, the study by Sarkar et al. (2011)[21] extended this theory by applying the Poisson distribution to the EPQ model, a variant of EOQ focused on production. Their model specifically addressed discrete demand i.e., demand measured in integers such as order counts or units and incorporated the concept of imperfect production, where a proportion of items may be defective. Brill and Chaouch's (1995)[22] study introduced a novel perspective by developing an EOQ model capable of handling randomly changing demand rates at unpredictable times. This model assumes that demand can shift between different states, such as two positive demand levels or zero demand based on a continuous-time Markov chain. These shifts may be triggered by real-world events, such as an economic recession reducing demand for luxury goods, or the resolution of labor disputes causing a sudden surge in raw material requirements. By incorporating these stochastic changes into inventory planning decisions, the model developed by Brill and

Chaouch enables managers to adjust order quantities flexibly, minimizing the risk of stockouts or excess inventory under complex scenarios.

In inventory management, modeling stochastic demand is critical for optimizing stock levels, order timing, and costs. The Poisson distribution is widely used, especially in EPQ models and systems with random demand. Named after Siméon Denis Poisson[23], this discrete probability distribution models the number of events in a fixed time or space interval, assuming independent events occur at a constant average rate (λ). In inventory, it represents customer demand, like daily orders or hourly sales, excelling for discrete, independent, low-frequency, or irregular demand. Per ResearchGate[24], Poisson suits low-demand cases (a few units yearly) or short product life cycles. Compared to normal, binomial, gamma, or Weibull distributions, Poisson is ideal as inventory demand is typically integer-based (units or orders). The normal distribution, being continuous, allows impractical real values (e.g., 2.5 units) and struggles with zero demand. Poisson's independence assumption fits random purchasing behavior (e.g., call center calls per minute). The binomial distribution, while discrete, requires a fixed number of trials (n), whereas demand has no such limit, making Poisson more flexible. The Poisson distribution requires only one parameter λ, representing the average constant demand rate, making it easy to estimate and integrate into mathematical models. In contrast, the normal distribution needs two parameters (mean and standard deviation), requiring more complex data estimation. Gamma and Weibull distributions, per ResearchGate[24], involve multiple parameters, unnecessarily increasing computational complexity for stable short-term demand. Poisson excels in modeling rare, random events, like a few daily orders, where the normal distribution's symmetric assumption fails to capture highly skewed or low-frequency demand. Poisson's simple formula enables easy integration into inventory optimization models like EPQ without adding computational complexity. Conversely, log-normal or Weibull distributions, though flexible, have complex formulations that complicate inventory optimization problems (ScienceDirect[25]). Poisson is preferred for its ability to model discrete, independent demand with a constant average rate, offering simplicity, practicality, and suitability for inventory management, especially for rare events.

2.2.3 Backorder policy in imperfect production systems

An imperfect production system is a practical model where some manufactured products are defective due to equipment wear, process errors, or human mistakes. These defective items are either recycled or discarded, raising production costs and making inventory management harder. Unlike traditional EOQ or EPQ models, which assume all products are perfect, this model reflects real-world conditions, preventing issues like stockouts or excess inventory.

The study by Sarkar et al. (2011)[21]created an EPQ model for an imperfect production system with variable demand. The defect rate increases during production, and defective items are recycled at extra cost. This reflects industries like electronics or automotive, where defects are common and recycling is key. The model uses probability distributions (e.g., uniform or Poisson) to simulate demand, fitting markets with unstable demand. It aims to maximize profit by finding the best production quantity (Q) and number of production cycles (n), balancing costs for production, storage, recycling, and shortages (if any). It shows that imperfect systems are both a challenge and a chance to improve management. The backorder policy is an inventory strategy that allows shortages. When demand exceeds supply, unfulfilled orders are recorded and filled later when stock is available. This differs from lost sales, where unmet demand is lost, hurting revenue and customer ties. In Sarkar's model, when inventory runs out, orders are backordered and fulfilled in the next batch. This helps maintain customer relationships and avoid lost sales but adds costs like delays or logistics. In imperfect systems, backorders are vital as defects can slow production, raising shortage risks. However, recycling defects takes time, increasing customer wait times and possibly affecting satisfaction. Thus, balancing backorder costs (waiting time and reputation) with production, storage, and recycling costs is key. The study highlights that the backorder policy is a management tool that must be optimized for imperfect systems. In summary, it reduces the risk of lost sales and maintains customer relationships, especially in competitive industries like retail or component manufacturing.

Implementing the backorder policy needs careful planning, as it can increase costs like customer waiting time or logistics and may lower satisfaction if delays are long. In Sarkar et al. (2011)[22], the EPQ model for an imperfect production system with variable demand used a backorder policy to handle shortages and maximize profit. This model offers a clear way to manage inventory and production in uncertain settings where demand and product quality vary. The results show that using the backorder policy with defective products and recycling improves inventory management theory, helping businesses make better production and inventory decisions. In the future, the model could include factors like lead time, transportation costs, or pricing strategies for a fuller view of supply chain management in imperfect systems. This would boost economic efficiency and help businesses handle real-world challenges like market changes and sustainability needs.

2.3 Research Gaps and Future Research Directions

Previous studies on inventory and production models, although having made considerable progress, still leave significant gaps in simultaneously integrating critical factors such as stochastic demand, imperfect production systems, backorder policies, and energy consumption. Specifically, the study by Sarkar et al. [21] developed an EPQ model with stochastic demand and a time-dependent defective rate but did not consider energy consumption in production or storage, thereby overlooking increasingly important sustainability requirements in modern supply chain management. Similarly, Zanoni et al. (2014)[15] incorporated energy consumption in both production and storage but assumed constant demand, failing to reflect the actual volatility of the market, where demand often follows probability distributions such as Poisson or uniform distributions.

H.-N. Nguyen et al. [26], focused on energy consumption in production and storage but ignored factors such as the defective rate or order shortages, while also assuming fixed or deterministic demand, reducing the practical applicability of the model in high-variability industries such as electronics, automotive, or fashion. Meanwhile, Sani (2023)[27] and Roy et al. [28] considered shortage factors and applied backorder policies in their models but did not integrate energy consumption or defective rates, making these models incapable of addressing both economic efficiency and environmental challenges simultaneously. For example, Sani [27] employed quadratic demand and a fixed defective rate but did not address energy, while Roy et al. [28] focused on stochastic demand and shortages without considering imperfect production systems.

Previous studies have not provided a unified model that addresses four key elements simultaneously. This gap is critical for modern production systems, which must balance economic profit with emission reduction and sustainability goals. Without a comprehensive model, existing studies lack practical relevance and limit businesses' ability to make effective inventory and production decisions. To fill this gap, this study introduces the EEPQ model, a more comprehensive and practical framework for imperfect production systems.

Table 2.1 Review of Integrated Inventory Models with Sustainability Considerations

Author	**Demand rate**	**Failure rate**	**Shortage**	**Energy implication**	
				Production	**Warehousing**
Sarkar et al. [29]	Stochastic	Variable	✓		
Widyadana & Wee [7]	Constant	Constant			
Zanoni et al. [15]	Constant	None		✓	✓
Pal et al. [30]	Random	Variable			
Roy et al. [28]	Stochastic	None	✓		
Manna et al. [31]	Dependent	None			
Kundu et al. [32]	Constant	Stochastic			
Abdel-Aleem et al. [33]	Constant	None			
Nezami & Heydar [18]	Constant	None		✓	✓
Dari & Sani [34]	Constant	Variable			
Karthick & Uthayakumar [35]	Stochastic	None			
H. N. Nguyen et al. [26]	Constant	None			
Sani [27]	Quadratic	Constant	✓		
H.-N. Nguyen et al. [36]	Constant	None		✓	
H.-N. Nguyen et al. [19]	Deterministic	None	✓	✓	✓
H.-N. Nguyen et al. [37]	Constant	None		✓	✓
This study	**Stochastic**	**Variable**	✓	✓	✓

The EEPQ model is built upon the integration of four core elements, simultaneously overcoming the limitations of previous studies and meeting real-world challenges in modern supply chain management:

1. **Stochastic demand**: EEPQ uses the Poisson distribution to simulate demand, reflecting the uncertainty and discreteness commonly observed in industries such as retail, component manufacturing, or food. Unlike the models by H.-N. Nguyen et al. [26] or Zanoni et al. [15], which assume constant demand, EEPQ captures market variability, enabling firms to plan production and inventory more flexibly.
2. **Imperfect production system**: The model considers a time-dependent defective rate, similar to the approach of Sarkar et al. [21], but further includes a recycling process for defective products with specific associated costs.
3. **Backorder policy**: EEPQ incorporates a backorder policy, allowing unfulfilled orders to be recorded and fulfilled once inventory is replenished, as in the models of Sani and Roy et al.[28].
4. **Energy consumption**: Unlike Sarkar et al. [21] or Sani[34], EEPQ integrates energy costs in both production (using the concept of Specific Energy Consumption – SEC) and storage, drawing inspiration from Zanoni et al. [15] and H.-N. Nguyen et al. [26].This ensures that the model optimizes not only economic profit but also minimizes environmental impact, aligning with sustainable development goals and carbon emission reduction requirements in the supply chain.

By integrating the above elements, EEPQ offers a powerful tool for optimizing expected profit in complex production systems where demand fluctuates, defects are frequent, and energy efficiency is increasingly critical.. Furthermore, the model not only fills an important research gap but also opens promising avenues for future work, such as analyzing the impact of lead time, transportation costs, or integrating green technologies like renewable energy into inventory management. These directions promise to further enhance both economic performance and sustainability, supporting businesses in adapting to a rapidly changing and environmentally demanding production landscape

CHAPTER 3: MATHEMATICAL MODEL

Chapter 3 focuses on the development and analysis of the mathematical model, providing a theoretical foundation for addressing issues in inventory and production management. It details the inventory management model, imperfect production systems, and the practical analysis of the Poisson distribution model in production management. Additionally, this chapter introduces the notation, including parameters and variables, along with core assumptions, particularly related to the EPQ model with imperfect production and energy consumption. This content establishes a critical foundation for developing practical and efficient solutions in production systems.

3.1 Problem description

Consider a production system in which a certain percentage of the total output is defective (non-quality products). The production system consistently generates defective products due to worker fatigue or wear and tear of machine parts during prolonged operation. The study examine a model where α_i represents the percentage of defective products, and the probability of defective products during the t_1-th time period is f_i. The production system is analyzed under the influence of energy consumption when there is only one item. The energy consumption will be calculated during the production and storage processes. Here, the production system starts with a production rate $P_{min} \leq P \leq P_{max}$ and continues until time t_1. If the production batch size is Q, then the production time is $t_1 = Q/P$. During the production period $[0, t_1]$ the inventory gradually increases after adjusting for demand. The accumulated inventory at time t_1 will decrease gradually and return to zero by time T. In this model, total demand d is uniformly distributed over the interval [0, T] The model also considers the case where demand exceeds production ($D = Pt_1 + 1$) which leads to shortages. The model examines demand that follows a discrete distribution $g(D)$. The discrete distribution considered here follows a Poisson function.

3.1.1 Inventory management

a. Practical issues in inventory management

Inventory management is essential to ensure smooth business operations and efficient response to market demand. First, inventory enables businesses to meet market needs in a timely manner, especially during peak seasons such as holidays or promotional campaigns. For instance, large retailers like Walmart maintain high inventory levels before events like Black Friday to prevent stockouts and ensure uninterrupted shopping experiences. According to a Deloitte (2021) [38] study, businesses that optimize inventory can reduce related operating costs by 20–30% and increase customer retention rates by up to 15% due to on-time product availability. Second, raw material inventory helps minimize production disruptions, ensuring continuous operation even when the supply chain is affected. During the COVID-19 pandemic, car manufacturers such as General Motors had to halt production due to a shortage of semiconductor chips, highlighting the importance of maintaining strategic component inventory.

Lastly, effective inventory management helps optimize costs by balancing storage costs (e.g., warehousing, preservation) and opportunity costs (e.g., lost sales due to stockouts). A well-designed inventory system not only reduces waste but also strengthens competitiveness by enabling fast and reliable product delivery, thus enhancing the firm's market position.

Despite its benefits, inventory management also faces many major challenges, especially in a volatile market and increasingly complex global supply chains. One of the biggest challenges is high storage costs, which can account for 15–40% of the value of goods annually, including warehouse rental, electricity, labor, and insurance. For perishable products such as food or pharmaceuticals, special preservation costs (e.g., cold storage) further increase financial burdens. For example, a frozen food company may spend millions of dollars annually to maintain storage temperatures below −18°C. According to a PwC (2020)[39] report, high warehousing costs are among the biggest barriers for small and medium-sized enterprises to scale up. Second, the risk of stockouts during periods of high demand can lead to lost sales and damage to brand reputation. A McKinsey (2020) study pointed out that 70% of customers will switch to competitors if the product they want is unavailable, especially in the retail and e-commerce sectors. This challenge becomes even more severe in the face of market volatility caused by factors such as raw material price fluctuations, trade policy changes, or logistics disruptions. For instance, the 2021 container shortage forced many businesses to increase safety stock, significantly raising costs. Lastly, obsolete goods are a major risk in industries with short product life cycles such as technology or fashion. Older smartphone models often need to be heavily discounted when new versions are released, causing financial losses if unsold in time. These challenges require businesses to adopt flexible and accurate inventory management strategies to minimize risk and optimize performance.

b. The EPQ Model (Economic Production Quantity) and Its Practical Applications

The EPQ model operates under the assumption that production rate (P) and demand (d) are constant, and that the business produces goods in batches of size (Q) over a specific period. Unlike the EOQ model, where inventory is received all at once, EPQ allows goods to be produced gradually and consumed during the production process. As a result, the maximum inventory level is lower than in EOQ, since goods are not accumulated all at once. This model ensures that a business produces the right quantity to minimize costs while maintaining a steady flow of goods to meet market demand.

- Practical Applications

The EPQ model is widely applied in continuous production industries where goods are produced and consumed simultaneously. For example, an electronic component manufacturer can use EPQ to determine the number of components to produce in each batch, ensuring inventory does not exceed necessary levels while still fulfilling customer orders. Similarly, in the packaging industry, EPQ helps optimize the production of cardboard boxes or plastic bottles, reducing storage costs and avoiding resource waste. To implement EPQ successfully, businesses should integrate the model with modern management tools such as ERP systems, build strong partnerships with suppliers, and use advanced demand forecasting methods to minimize risks. Inventory dynamics modeling based on the EPQ model provides a mathematical approach for analyzing and forecasting inventory levels throughout a production cycle. The model is built upon the following assumptions and formulas, helping businesses plan production more effectively.

- Detailed analysis

The EPQ model provides key benefits for manufacturers. First, it lowers holding costs by finding the optimal production quantity, reducing average inventory levels. This is especially helpful for industries with standard goods, where storage costs are high. A 2019 Harvard Business Review study found that optimized models like EPQ can cut storage costs by 50–70% compared to traditional methods. Second, EPQ boosts production efficiency by keeping the production line running smoothly without interruptions from material shortages or excess inventory. Finally, it improves planning accuracy with a clear mathematical framework, helping businesses predict production cycles and adapt to market demand. However, EPQ has practical challenges. First, it needs accurate data on demand (D), production rate (P), and costs (S, h). Inaccurate demand forecasts or production disruptions can lead to stockouts or overstocking. For instance, during the COVID-19 pandemic, volatile demand reduced EPQ's effectiveness. Second, EPQ depends on a stable supply chain for timely raw material delivery. Any disruption, like logistics issues or supplier errors, can affect production schedules. Lastly, implementing EPQ requires investment in technology, such as ERP software, and staff training, which can be costly. Compared to Just-In-Time (JIT), EPQ emphasizes cost optimization through calculations, while JIT focuses on minimizing inventory. JIT needs tight supply chain coordination and accurate forecasts, but EPQ is more flexible, handling larger production batches and suiting businesses with less stable supply chains. Both aim to cut waste and boost efficiency and can be combined in a broader inventory strategy.

3.1.2 Imperfect Production Systems

a. Practicality of Imperfect Production Systems

Imperfect production systems, as described in Sarkar et al. (2011), reflect the real-world condition that a certain proportion of products (typically around 10%) fail to meet quality standards due to factors such as prolonged machine usage, technical faults, or human management errors. The practicality of implementing this model in production management is evident in three main aspects: cost control, process optimization, and brand reputation enhancement. Defective products result in significant costs related to repair, replacement, or disposal. According to Sarkar et al. (2011), the rework cost (denoted C_0) is integrated into the expected profit function, allowing businesses to estimate and manage costs associated with defective products. The expected number of defective products is calculated using the formula $\sum_{i=1}^{n} \alpha_i f_i$ which combines the defect rate (α_i) and the defect probability (f_i) to estimate the average number of defects over n periods. This enables firms to plan budgets effectively, allocate resources for rework activities or alternative raw materials, and minimize financial losses. For example, in Sarkar et al. (2011)'s Case Study 2, with parameters such as C_0 = \$30 and a defect rate increasing from 0.05 to 0.35 over 7 days, the model determines the optimal production quantity (Q^* = 15 units) to maximize profit (\$575.6254), demonstrating its practical application in balancing costs and output. This is the prediction table of defect rates and defect probabilities converted according to h, as presented in Sarkar et al. [29].

Moreover, the model allows businesses to estimate holding costs (C_k) and shortage costs (C_s), especially under stochastic demand conditions that may result in stockouts. By optimizing the production lot size (Q), businesses can minimize total cost and ensure economic efficiency in an imperfect production environment.

b. Production Process Optimization

The application of quality management methods such as Six Sigma or Lean Manufacturing, as previously discussed, can be supported by data from this model. For example, if α_i increases significantly at a specific stage, businesses may implement corrective actions such as employee retraining, machine part replacement, or process adjustments to reduce the defect probability (f_i). The numerical examples in Sarkar et al. [29] illustrate this: in the Poisson distribution case (Example 2), with given α_i and f_i values, the model determines the optimal number of production days (n^*= 5) to maximize profit, showing its applicability to short-term production planning.

In competitive markets, product quality is a critical factor for maintaining customer trust and brand reputation. Sarkar et al. [29] emphasized that defective products should be reworked to restore original quality, thereby preserving brand image in oligopolistic markets. Reducing the defect rate through data-driven interventions not only improves product quality but also enhances customer satisfaction. The EPQ model of Sarkar et al. [29] enables firms to balance rework costs and expected profits, ensuring defective products are managed efficiently without significantly increasing total costs. Although the model by Sarkar et al. [29] offers high practical value, it still presents some limitations. First, the model assumes zero lead time, which is unrealistic in many industries, especially those with complex supply chains. Second, the fixed production rate (P) may not suit flexible production systems where the rate varies with market demand. Lastly, collecting real-time data to determine α_i and f_i requires substantial investment in technology and statistical analytics software, which may be a barrier for small and medium-sized enterprises.

3.1.3 Analysis of the Practicality of the Poisson Distribution Model in Production Management

a. Overview and Applications of the Poisson Distribution Model

The Poisson distribution model is a fundamental mathematical tool used to describe random, independent events occurring within a fixed time or space interval. This model is particularly suitable in fields such as retail, manufacturing, and logistics, where market demand fluctuates unpredictably. According to Sarkar et al. [29], the Poisson distribution is commonly applied to simulate stochastic demand in the Economic Production Quantity (EPQ) model, especially in imperfect production systems.

The probability formula of the Poisson distribution is defined as:

$$P(D = k) = \frac{e^{-\lambda}\lambda^k}{k!} \tag{11}$$

Where:

- D is the demand within a time interval T
- λ is the expected value (average number of units demanded)
- k is the actual number of units demanded
- $e^{-\lambda}$ is the normalization constant

An important factor in production management is the probability of shortage, calculated using the formula:

$$P(D > Pt_1) = 1 - \sum_{k=0}^{Pt_1} \frac{e^{-\lambda}\lambda^D}{k!} \tag{12}$$

This formula enables firms to quantify the risk of failing to meet demand when actual demand exceeds production capacity. As a result, it supports decisions regarding optimal production quantity and safety stock levels. The Poisson distribution model proves to be effective in situations involving random demand, such as order volumes during peak shopping seasons (e.g., Black Friday, Lunar New Year) or new product launches. For example, an electronics retail chain preparing for the end-of-year holiday season can use the Poisson distribution to forecast smartphone demand.

Assume historical data shows an average of 50 orders per day (λ = 50) during the holiday week. If the company only produces or stocks 40 units per day (Pt_1= 40), it can calculate the probability of shortage using the formula:

$$P(D > 40) = 1 - \sum_{k=0}^{40} \frac{e^{-50}50^k}{k!}$$

Using software tools such as Python or Mathematica, the result yields:

$$P(D > 40) \approx 0.841$$

This means there is approximately an **84.1% probability** that demand will exceed supply capacity. Based on this probability, the business may decide to increase production or stock an additional 20 units as **safety inventory** to reduce the shortage risk to an acceptable level e.g., under 10%.

This model supports strategic decision-making by helping businesses:

- Optimize resource allocation
- Anticipate and respond to market fluctuations
- Minimize risks in supply chain management

By using the Poisson distribution in this way, businesses can proactively balance supply and demand during high-variability periods, improving service levels and customer satisfaction.

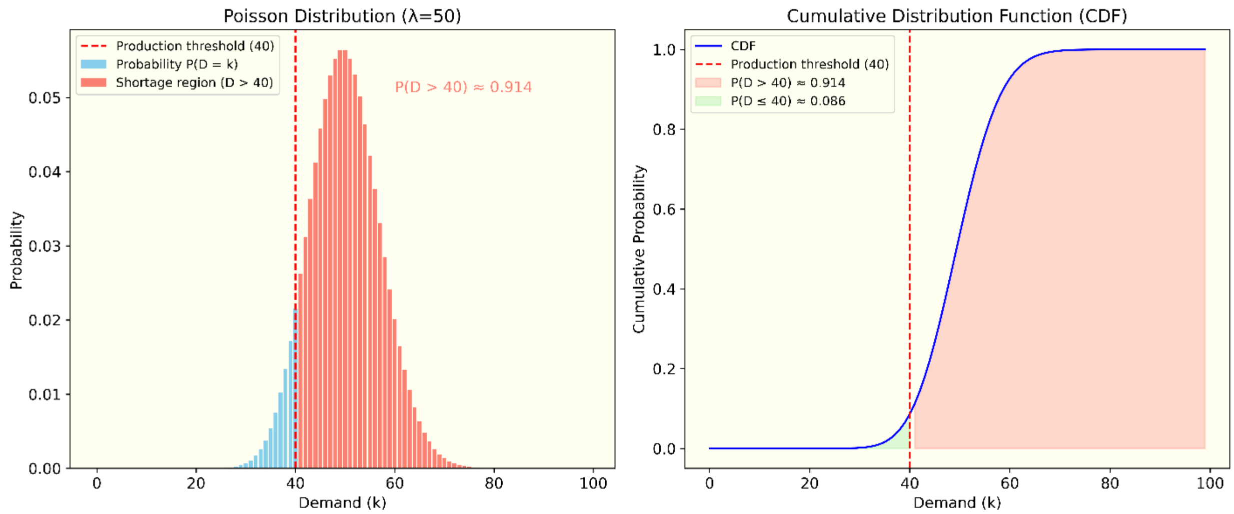


Figure 3.1 Poisson Distribution and Shortage Probability

Left Chart – Poisson Probability Distribution (PMF)

- A histogram representing the Poisson probability mass function with λ = 50.
- The horizontal axis (x-axis) shows the demand quantity k, ranging from 0 to 100 units.
- The vertical axis (y-axis) displays the probability P(D=k)
- The peak of the histogram is centered around k = 50, indicating the expected value of demand.
- The shape is skewed slightly to the right, typical of a Poisson distribution when λ is large.

Right Chart – Cumulative Distribution Function (CDF)

- A line plot showing the cumulative probability P(D≤k) for the Poisson distribution with λ = 50. The horizontal axis (x-axis) is k (demand units), and the vertical axis (y-axis) is the cumulative probability P(D≤k)).
- At k = 40, representing the production capacity threshold $Pt_1 = 40$.
- The area to the right of this vertical line (i.e., where D > 40) is shaded, highlighting the shortage probability P(D>40)≈0.841. This shaded region visually represents the risk of unmet demand, guiding the decision to increase production or add safety stock.

b. Practical Applications of the Poisson Distribution Model

The Poisson distribution model is a valuable tool for optimizing supply chains and production in industries like retail, manufacturing, and logistics, where demand varies. In sectors like food or fashion with short product life cycles, it helps reduce production or inventory when shortage risks are low, minimizing waste and storage costs. For example, Figure 3.1 shows a Poisson distribution with λ = 50 and a shortage probability of 0.841 at Pt_1 = 40, aiding managers in visualizing risks and making informed production choices. As noted on April 22, 2025, this model excels during peak times like Lunar New Year, helping supermarkets plan inventory to prevent shortages. However, the model has limitations. First, it assumes a constant λ (expected demand), but λ can change due to marketing, price shifts, or trends. For instance, a promotion might raise λ from 50 to 80 orders daily, making fixed-λ forecasts unreliable. Second, it overlooks factors like lead time, shortage costs (lost revenue, compensation), or logistical limits like warehouse capacity. Sarkar et al. (2011) note that assuming zero lead time reduces its use in complex supply chains. Third, λ relies on historical data, which may be unavailable or unreliable for new markets or products.

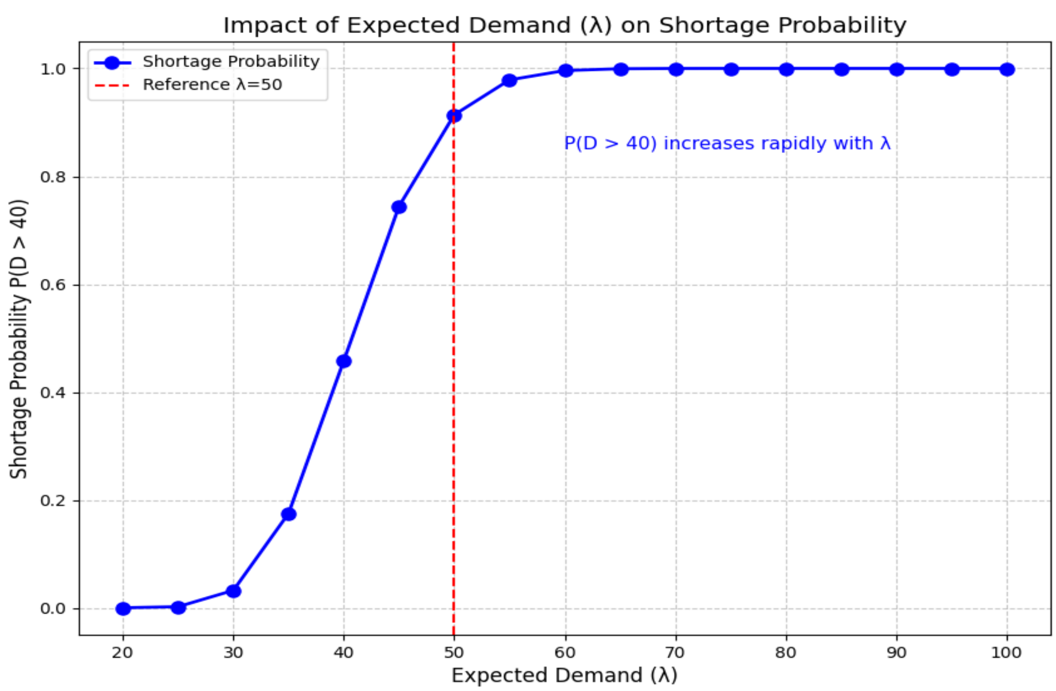


Figure 3.2 Impact of expected demand on shortage probability

Figure 3.2 shows the shortage probability P(D > 40) as a function of λ (ranging from 20 to 100) (Pt_1 = 40), illustrating how the shortage probability increases rapidly as λ rises, emphasizing the model's sensitivity to λ. To address these issues, businesses can use data analytics technologies such as ERP software or artificial intelligence (AI). A retailer, for example, could use AI to analyze sales data from past holiday seasons to forecast a dynamic λ by day, allowing for flexible production planning.Integrating additional elements like shortage cost into the model also increases its practical value and combining AI and the Just-In-Time (JIT) approach helps overcome the limitations of a fixed λ, enhancing accuracy in supply chain management.

3.2 Notation

This notations are used for mathematical modeling of the study:

3.2.1 Parameters

a. Production Setup Parameters

This group includes parameters related to the setup and operational aspects of the production process, such as initiating production runs and managing demand over a cycle.

- S: setup cost ($/setup)

The setup cost represents the fixed expense incurred each time a production run is initiated, covering costs such as machine calibration, labor, or preparatory materials.

- T: cycle time (h)

Cycle time is the duration from the start of one production run to the start of the next, encompassing both production and non-production periods. This parameter determines production frequency, impacting holding costs, setup costs, and demand fulfillment.

- D the demand over the period [0, T] which is uniform over the period (unit)

Demand D denotes the total units of product required over the cycle time T , assumed to be uniformly distributed, resulting in a constant demand rate $\frac{D}{T}$. This parameter serves as the basis for production and inventory planning, guiding decisions on production quantities and inventory levels to meet demand without excess or shortages.

- $g(D)$: The model examines demand that follows a discrete distribution $g(D)$. The discrete distribution considered here follows a Poisson function.

b. Energy Consumption Parameters

This group includes parameters related to the energy used in the production process, which is critical for cost management and sustainability.

- W: idle energy consumption (kW)

  Idle energy consumption is the power used by the production system when it is not actively producing, such as during standby periods. This parameter represents a baseline energy cost incurred regardless of output.

- k: energy consumption for one unit producing (kWh/unit)

  The parameter k indicates the energy required to produce a single unit of product, reflecting the energy efficiency of the production process. It directly links production volume to energy costs and is used to compute production-related energy expenses.

- E: energy cost ($/kWh)

  Energy cost is the price per unit of energy consumed, converting energy usage (both idle and production-related) into monetary costs. This parameter connects energy consumption to operating expenses, appearing in the total cost function to assess the financial impact of production decisions.

c. Quality and Defect Parameters

This group includes parameters related to defective items and their repair costs, which affect product quality and cost management.

- α_i: percentage of defective items at the ith time with probability f_i

  The percentage of defective items α_i represents the proportion of substandard products at a specific time or stage, with probability f_i reflecting the variability of defect rates. This parameter reduces effective production yield, increasing costs for repair or disposal, and is used to calculate expected defective units. It supports quality control planning and risk assessment in production.

- C_0: repairing cost per defective item ($/(unit.h))

  The repairing cost C_0 is the expense to fix a defective item, measured per unit and per hour of repair time. This parameter quantifies the financial impact of defects, contributing to the total cost function and incentivizing quality improvements to reduce defect rates. It also aids in evaluating whether repairing or preventing defects is more cost-effect *d. Inventory Cost Parameters*

This group includes parameters related to defective items and their repair costs, which affect product quality and cost management.

- C_h: holding cost per unit per unit time ($/(unit.h))

The holding cost per unit per unit time, denoted (C_h), represents the expense incurred for storing one unit of inventory for one unit of time, typically measured in dollars per unit per hour. This cost includes expenses such as warehousing, insurance, spoilage, obsolescence, and capital tied up in inventory. It also reflects the trade-off between overstocking and the risk of shortages, making it a pivotal parameter in achieving cost efficiency and operational agility.

- S_h: shortage cost per unit per unit time ($/(unit.h))

The shortage cost per unit per unit time, denoted (S_h), is the cost incurred when demand exceeds available inventory, resulting in a stockout, measured in dollars per unit per hour of shortage. Practically, (S_h) influences strategic decisions, such as investing in demand forecasting or flexible production systems to mitigate stockouts. It also highlights the importance of aligning production capacity with demand patterns, making it a critical parameter for ensuring customer satisfaction and financial stability.

e. Pricing and Profit Parameters

This group includes parameters related to the financial aspects of selling products and determining profitability, which are essential for pricing strategies and revenue optimization in a market-driven context.

- S_p: profit per unit item = (selling price – purchasing cost) of one unit ($/unit)

The profit per unit item, denoted (S_p), represents the difference between the selling price and the purchasing (or production) cost of a single unit, measured in dollars per unit. This parameter encapsulates the net financial gain from each unit sold, serving as a cornerstone for assessing the profitability of production and sales activities. μ: The market absorption rate affects prices

The market absorption rate, denoted μ, represents the rate at which the market accepts or purchases the product, influencing the pricing strategy and demand dynamics. This parameter reflects how quickly products are sold, which in turn affects the selling price and overall revenue. In economic and inventory models, μ is often modeled as a factor that adjusts prices based on market conditions: a high absorption rate may allow for higher prices due to strong demand, while a low rate may necessitate price reductions to stimulate sales.

f. Warehouse Temperature Parameters

This group includes parameters related to the temperature inside and outside the warehouse, which are critical for managing energy consumption and maintaining optimal storage conditions for goods. These parameters support the optimization of warehouse operating costs, particularly in systems requiring strict temperature control, such as cold storage or pharmaceutical warehouses.

- T_W: Expected warehouse temperature (°C)

The expected warehouse temperature, denoted T_W, is the target temperature set to ensure optimal storage conditions for goods, measured in degrees Celsius. This parameter represents the ideal temperature inside the warehouse, such as -18°C for frozen foods or 2-8°C for pharmaceuticals, and serves as a cornerstone for determining the energy required for cooling or heating the warehouse. In warehouse management models, T_W is used to calculate the heat load and energy costs, often appearing in objective functions that optimize operating expenses.

- T_r: Referenced warehouse temperature (°C)

The referenced warehouse temperature, denoted T_r, is the standard temperature used for comparison or normalization in energy calculations, measured in degrees Celsius. This parameter reflects a fixed benchmark, often the average or ideal temperature of the warehouse under standard conditions, serving as a reference point to assess the temperature deviation (T_W relative to T_r). T_{hot}: Outside warehouse temperature (°C)

The outside warehouse temperature, denoted T_{hot}, is the ambient temperature surrounding the warehouse, typically the outdoor air temperature, measured in degrees Celsius. This parameter represents the environmental impact on the warehouse's heat load, directly affecting the energy required to maintain T_W. In optimization models, T_{hot} is used to calculate cooling or heating energy, often appearing in functions like energy = f(T_{hot}-T_W, *ρ, α, β).* The value of T_{hot} varies with weather conditions and geographic location, making it a key factor for assessing warehouse operating costs across different scenarios.

g. Warehouse Storage Coefficients

This group includes coefficients related to the physical characteristics and fill level of the warehouse, influencing specific energy consumption (SEC) and operational efficiency. These coefficients support the optimization of energy costs and warehouse management in production and storage systems.

- ρ: Coefficient linking SEC to various storage temperatures

The coefficient ρ links specific energy consumption (SEC, measured in kWh/kg or kWh/m³) to storage temperature conditions, such as T_W, T_r, and T_{hot}. This parameter reflects the extent to which temperature differences affect the energy required to maintain the warehouse environment, serving as a cornerstone for evaluating operating costs. In energy models, ρ appears in formulas like $SEC = \rho \times f(T_{hot} - T_W)$, adjusting energy costs based on temperature conditions. The value of ρ depends on warehouse design, cooling system efficiency, and goods characteristics, making it a critical indicator for comparing operational scenarios. Practically, ρ guides energy optimization strategies, such as adjusting T_W or upgrading insulation to reduce SEC. It also interacts with temperature parameters, as a large difference between T_{hot} and T_W increases the effective value of ρ, leading to higher energy costs.

- α, β: Positive coefficients dependent on the characteristics of the warehouse, where β ∈ (0, 1)

The coefficients α and β are positive parameters describing the physical and operational characteristics of the warehouse, such as insulation, size, or cooling/heating system efficiency, with β constrained to the interval (0, 1). The coefficient α typically represents the fixed or baseline energy consumption of the warehouse, independent of fill level, while β reflects diminishing efficiency as factors like temperature or warehouse load change. In energy models, α and β appear in functions like energy = α + β × f(T_{hot}- T_W, ρ), adjusting costs based on warehouse structure. The values of α and β depend on warehouse design and materials, making them critical for evaluating operational efficiency. Practically, α and β inform investment decisions, such as using better insulation or more efficient cooling systems, to reduce energy costs. They also interact with ρ, as improved warehouse design can reduce the effective value of ρ, leading to lower SEC. These parameters emphasize the need to optimize warehouse infrastructure to achieve energy and cost efficiency.

- *δ, γ:* Positive coefficients dependent on the filling level of the warehouse

The coefficients δ and γ are positive parameters reflecting the impact of the warehouse fill level (the ratio of stored goods to maximum capacity) on energy consumption. The coefficient δ typically represents additional energy costs as the warehouse becomes fuller, due to the need to cool/heat more goods, while γ reflects changes in energy efficiency based on fill level. In energy models, δ and γ appear in functions like *SEC = f(δ × fill level, γ × heat load)*, adjusting costs based on stored inventory. The values of δ and γ depend on goods type and warehouse design, making them critical for optimizing inventory management. Practically, δ and γ guide storage strategies, such as maintaining an optimal fill level to reduce energy costs or adjusting intake/output schedules to avoid overloading. They also interact with parameters like ρ and α, as higher fill levels increase heat load, leading to higher energy costs. These parameters underscore the balance between storage capacity and energy efficiency, requiring businesses to manage inventory strategically to optimize costs.

3.2.2 Variable

a. Dependent variable

Below is a detailed explanation of each dependent variable in the integrated energy-optimized inventory model (EEPQ_S), highlighting their roles and significance in the production and warehousing system. The explanations are presented in clear, structured paragraphs without using formulas.

- Q: production lot size (unit)

The production lot size, or Q, represents the number of products manufactured in a single production cycle. This variable determines the scale of each production batch, directly affecting the amount of goods sent to the warehouse to meet market demand. A larger lot size can reduce the frequency of machine setups, thereby saving setup costs, but it increases warehousing costs due to higher inventory levels. Conversely, a smaller lot size lowers warehousing costs but may raise production costs due to more frequent machine startups. Thus, Q is a critical factor in balancing production and inventory costs, playing a pivotal role in optimizing the overall economic efficiency of the system.

- I_{1max}: maximum storage capacity of the warehouse when there is no shortage (unit)

The maximum warehouse storage capacity with no shortages, or $I_{1_{max}}$, is the highest inventory level the warehouse can hold when production fully meets demand, avoiding any stockouts. This variable reflects the warehouse's optimal storage capability under ideal operating conditions, where production is sufficient to fulfill demand without delays or additional supply. $I_{1_{max}}$ enables managers to assess the warehouse's capacity, ensuring sufficient space to store goods during periods of stable demand. It supports efficient storage planning, minimizes space wastage, and optimizes costs related to inventory management.

- $I_{2_{max}}$: Maximum storage capacity of the warehouse t when there is shortage (unit)
- I_1: Inventory level at time t when there is no shortage (unit)

The inventory level I_1 represents the amount of goods in the warehouse at a specific time t when production fully meets demand, with no shortages. This variable is used to monitor the warehouse's status under ideal operating conditions, where production is sufficient to fulfill all orders without interruption. I_1 helps managers track the actual inventory level during stable periods, facilitating effective production and warehouse management planning. It is particularly crucial for optimizing warehousing costs, as it reflects the inventory needed to maintain a continuous flow of goods without wasting resources.

- I_2: Inventory level at time t when there is shortage (unit)

The inventory level I_2 indicates the amount of goods in the warehouse at a specific time t when demand exceeds production capacity, leading to temporary shortages. This variable applies when the system cannot immediately meet all demand, causing inventory levels to drop significantly or even reach zero in some periods. I_2 helps assess the warehouse's status under challenging conditions, supporting decisions like ramping up production, managing delayed orders, or applying backordering policies. While fundamentally similar to I_1 in measuring inventory at time t, I_2 focuses on shortage scenarios, reflecting the warehouse's state when the system faces pressure from fluctuating demand or production constraints.

- $I_{(t)}$: inventory level at time t (unit)

The inventory level $I_{(t)}$ is a general variable representing the amount of goods in the warehouse at any given time t, encompassing both scenarios with and without shortages.

This variable provides a comprehensive view of the warehouse's status, allowing managers to track inventory changes over time under all operating conditions. $I_{(t)}$ serves as a universal indicator, aiding in analyzing inventory trends, evaluating the effectiveness of production and storage strategies, and making optimal decisions to balance inventory costs with demand fulfillment. It is particularly valuable for simulating system dynamics and ensuring flexibility in management.

- t_2: time of non-production time when there is shortage (h)

The non-production time t_2 is the duration within a production cycle when the system halts production due to depleted inventory during a shortage scenario. This variable occurs when demand exceeds supply, leaving the warehouse with no goods to meet orders, resulting in production interruptions. t_2 helps identify periods when the system is in a waiting state, supporting planning efforts to minimize downtime, such as increasing prior production or adjusting schedules. It is critical for managing opportunity costs and ensuring rapid system recovery during high-demand periods, maintaining continuity in the supply chain.

- t_3: time of production sub-time in shortage period (h)

The production sub-time t_3 is the additional time during a shortage period when the system resumes production to replenish missing inventory, fulfilling delayed orders. This variable appears after the non-production period (t_2), when the system restarts production to restock the warehouse and meet unmet demand. t_3 aids in assessing the time required to recover from shortages, helping managers optimize production schedules, minimize costs related to delayed orders, and ensure supply continuity. It is particularly vital in volatile demand scenarios, where rapid recovery is key to maintaining customer satisfaction and operational efficiency.

b. Decision variable

Below is a detailed explanation emphasizing the role and significance of each decision variable in the production and warehousing system of the integrated energy-optimized inventory model

- P: production rate (unit/h)

The production rate (P), measured in units per hour, shows how many products are made per unit of time. It affects how quickly a production batch is completed and how many goods are sent to the warehouse to meet demand. A high P speeds up production, meeting demand faster, but increases energy costs due to intense machine use and may raise defect rates from equipment stress or errors. A low P saves energy and reduces defects but lengthens production time, increasing inventory costs due to longer storage. Thus, P is a key variable for managers to balance production costs, energy costs, defect rates, and demand fulfillment, optimizing the system's economic and operational efficiency.

- t_1: time of production time (h)

Production time (t_1) is the duration of production activities in a cycle to create a batch of products. It affects how many goods are produced and when they're ready for warehousing or distribution. A longer t_1 produces more goods, ideal for high demand, but raises production and energy costs due to extended machine use. It also increases defect risks from worker fatigue, machine wear, or errors, leading to higher repair or rework costs, which hurts efficiency. A shorter t_1 cuts costs, energy use, and defects but may not meet demand, risking shortages. Thus, t_1 is a critical variable for managers to optimize production schedules, balance demand, control costs, and reduce defects, improving the efficiency of the production and inventory system.

3.3 Assumptions

3.3.1 EPQ model with imperfect production and energy consumption

The Modified Economic Production Quantity (EPQ) model is developed to optimize costs in imperfect production systems, where defective items, fluctuating demand, and adjustable production rates are prevalent. Unlike the traditional EPQ model, which assumes perfect production, this model incorporates real-world complexities such as defective item rates, discrete demand distributions, fully backlogged shortages, and machine idle states. It aims to minimize the expected total cost, including setup, holding, shortage, rework, and energy costs, making it particularly suitable for industries like electronics, textiles, or food processing. The following analysis examines the model's key parameters, evaluates its real-world applicability, and integrates the energy factor in warehousing, production, and machine idle states, drawing inspiration from Zanoni et al. (2014) on energy costs in production.

Defective item rate (α_i, f_i)The defective item rate, denoted α_i, represents the percentage of non-conforming units in a production lot, with the probability of defects during the t_1-th time period given by f_i, measured as a probability. This parameter reflects the imperfect quality of the production process, where defective items are reworked, or, if irreparable, incur a fixed deterioration cost C_0 (accounting for salvage value). In the modified EPQ model, α_i reduces the number of good units (1 - α_i)Q, increasing rework costs and deterioration costs, while extending production time to meet demand. It appears in the expected total cost function, influencing the optimal lot size $Q*$ and production rate $P*$. Practically, C_0 guides quality control strategies, such as investing in precision machinery to reduce defects in electronics or improving processes in textiles to minimize fabric flaws. It interacts with energy costs, as defective items require additional production, increasing energy consumption *(W/P)*. This parameter underscores the importance of balancing quality costs and production efficiency, requiring businesses to optimize α_i through technology and process improvements.

Adjustable production rate *(P).* The production rate, denoted P, is the number of units produced per unit time, adjustable within the range $P_{min} \leq P \leq P_{max}$, but only changeable at the start of a production cycle. This parameter represents flexibility in production management, affecting production time *(Q/P),* energy costs, and inventory levels. In the modified EPQ model, a high P reduces production time but increases energy costs *(e × W/P),* while a low P extends cycles, raising holding costs. P appears in the total cost function, impacting $Q*$ *and* $P*$ by balancing setup, holding, and energy costs. Practically, P informs production decisions in industries like plastics or metals, where machines operate in multiple modes. It interacts with α_i, as high defect rates require additional production, increasing time and energy, and with the idle state, as a low P extends non-production periods, incurring idle costs. This parameter emphasizes the need to optimize machine operations to minimize energy costs and meet demand efficiently. And the machine idle state refers to the energy cost, in monetary units per unit of time, when the machine is on but not producing. This is common in industries like steel or chemicals, where machines use energy in standby mode to maintain temperature or readiness. In the modified EPQ model, idle state occurs during non-production periods, calculated as *(1 - Q/P) × cycle*, impacting total costs. It encourages larger production lots to reduce idle frequency, but this increases holding costs. In practice, it guides scheduling, like in plastic injection molding, where idle machines still consume energy. It interacts with production rate (P), as a lower P lengthens cycles and idle time, and with warehousing energy, as less idle time may lead to larger inventories. This parameter highlights the need to manage machine states to optimize energy costs and production efficiency. Discrete demand, denoted $g(D)$, follows a Poisson distribution, representing variability in customer demand. This parameter captures market uncertainty, unlike the constant demand in the traditional EPQ, affecting inventory and shortages. In the model, $g(D)$ increases the risk of shortages, incurring a cost S_h per backlogged unit. It appears in the total profit function, complicating the derivation of Q^* *and* t_1 *and* P^* due to the need for numerical or simulation methods. Practically, $g(D)$ suits industries like retail or food, where demand fluctuates seasonally or due to events, aiding inventory planning and sales forecasting. It interacts with α_i, as defects reduce good units, increasing shortage risks during high demand, and with S_h, as demand variability drives backlogging. This parameter highlights the need for flexible inventory management to balance holding and shortage costs in volatile markets.

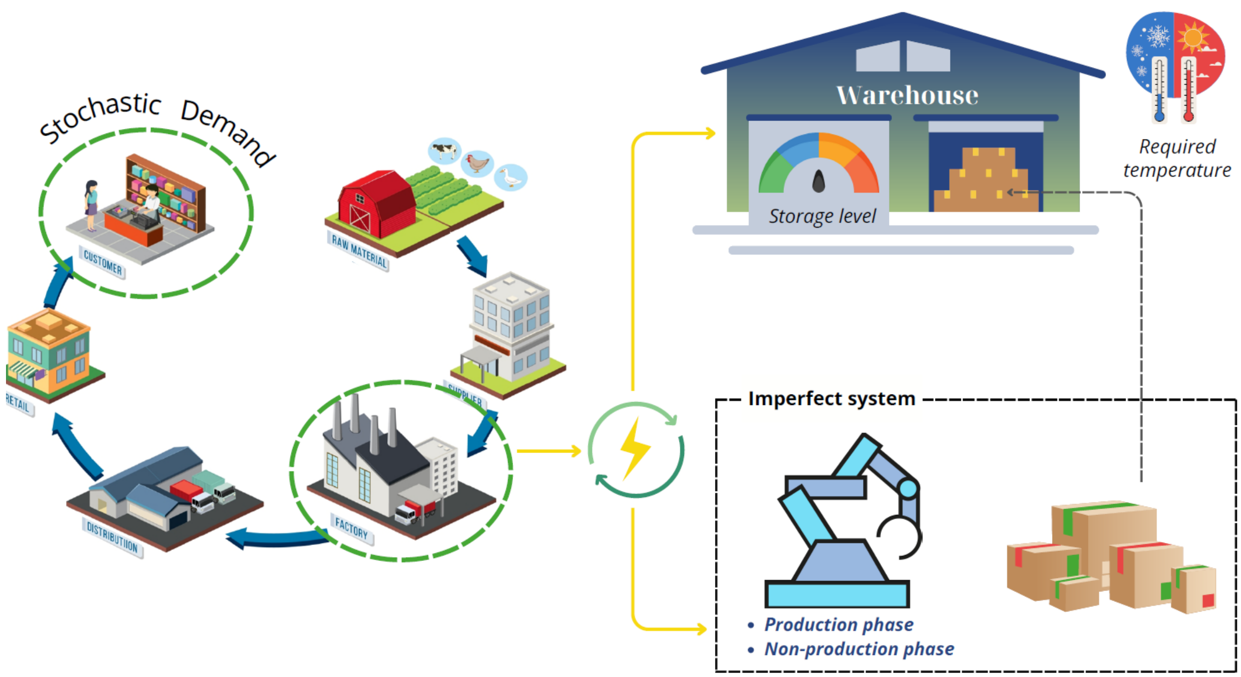


Figure 3.3 Contributions of studying for manufacturing practical

Shortages and backlogging. The shortage cost, denoted S_h, is the cost per unit of unmet demand, fully backlogged for later fulfillment. This parameter represents flexibility in meeting demand, allowing businesses to delay deliveries when inventory is insufficient, but incurring S_h based on the expected number of backlogged units (BS). In the modified EPQ model, S_h influences the total cost function, encouraging safety stock to reduce shortages but increasing holding costs h. Practically, S_h supports industries with make-to-order systems, like automotive or machinery, where customers tolerate delivery delays. It interacts with $g(D)$, as demand variability increases BS, and with α_i, as defects reduce good units, heightening shortage risks. This parameter underscores the balance between inventory and customer service, requiring businesses to optimize $Q*$ to minimize combined holding and shortage costs.

The modified EPQ model is highly applicable to real-world production with imperfect quality and variable demand. The defective item rate (α_i, f_i) fits industries like electronics (circuit defects) or textiles (fabric flaws), encouraging investment in quality control to lower holding (C_h) and ordering (C_o) costs. Poisson demand $g(D)$models sales variability in retail or food, aiding flexible inventory planning to prevent overstock or shortages. Allowing shortages suits make-to-order industries like automotive, where backlogging adds flexibility compared to traditional EPQ. Adjustable production rate *(P)* works for multi-mode factories (e.g., plastics), affecting energy and quality. Idle state costs reflect energy expenses in steel or chemical plants, promoting optimized scheduling. However, the model's complexity, driven by $g(D)$, requires numerical methods, challenging for small businesses. Assumptions like no in-cycle repairs and zero lead time may not fit industries with continuous repairs (e.g., automotive). The model highlights integrating quality, demand, and energy for effective production management. Energy is crucial in the modified EPQ model, impacting costs in warehousing, production, and idle states, especially with rising energy prices. In warehousing, energy costs arise from maintaining conditions (e.g., cold storage), depending on temperature differences and a specific energy consumption coefficient. Higher inventory levels increase cooling needs, raising holding costs. In production, energy use ties to production rate: a higher rate consumes more energy but shortens production time, while a lower rate saves energy but increases inventory costs. Defective products require extra production, adding energy costs. In idle states, standby energy costs accrue (e.g., in steel manufacturing), but a switch on/off policy can reduce them. Practical strategies include optimizing warehouse temperatures, adjusting inventory levels, using lower production rates during high energy price periods, or turning off machines. Energy interacts with defect rates and demand variability, as these extend production and inventory periods, increasing energy use. Integrating energy management drives equipment upgrades, better scheduling, and efficient inventory, boosting operational efficiency.

CHAPTER 4: MODEL DEVELOPMENT

Chapter 4 builds upon the theoretical foundations established in Chapter 3, focusing on the development of the EEPQ model under two primary scenarios: without shortages and with shortages (EEPQ_S). This chapter details the cost trust function, revenue function, and objective function for each scenario, providing a mathematical basis for optimizing production management. By analyzing both practical and theoretical aspects of these models, Chapter 4 plays a crucial role in formulating effective strategies to address challenges in modern production systems.

Many real-world problems are discrete, occurring in distinct, countable intervals rather than continuously. Studying our model with discrete demand distributions is crucial for practical applications. We analyze how discrete factors affect demand, using statistical methods to predict outcomes accurately. This helps build precise models, optimize production and energy costs, and manage imperfect production systems, where defects arise from human errors, machine wear, or external factors, unlike the perfect-quality assumption in the traditional EPQ model. Here, we consider a model where α_i represents the percentage of defective products, with the probability of defective products in the n$(t_1)^{\text{th}}$ period being f_i. Therefore, the expected total number of defective products is $\sum_{i=1}^{t_1} \alpha_i f_i$ and the lot size Q = $t_1 P$ for the n$^{\text{th}}$ period.

And the model examines demand that follows a discrete distribution g(D). The discrete distribution considered here follows a Poisson function.

$$g(D) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{D}}{D!} \; ; \; \lambda > 0; \; D = 0; 1; 2; 3; \ldots . +\infty \\ 0 \; ; \; Otherwise \end{cases} \tag{13}$$

4.1 Scenario without Shortages (EEPQ)

In the EEPQ model with no shortages, the system ensures enough stock to meet all demand. Goods produced, based on production time and rate, cover all demand levels, from low to high. Since demand is random, the model considers all scenarios, from zero to the factory’s maximum capacity. A probability sum calculates the likelihood of each demand level within this range, avoiding scenarios beyond production capacity that would cause shortages. This keeps the model realistic. For example, when calculating holding costs, it estimates average leftover stock by weighing each demand level's probability, ensuring cost and profit calculations reflect market fluctuations for better managerial decisions.

The probability sum in the EEPQ model is a crucial tool for production management, beyond just calculations ($\sum_{D=0}^{Q}(cost\ function)g(D) = \sum_{D=0}^{t_1P}(cost\ function)g(D)$. Market demand fluctuates constantly, from quiet days to peak times. With the EEPQ model's no-shortage rule, it ensures businesses can meet all demand within production capacity. Accounting for all demand levels, from zero to the production limit, is vital for accurate forecasting and planning. Limiting the probability sum to this range reflects the reality that a factory can only produce a fixed amount in a given time. If demand exceeds this, shortages occur, which the model avoids. The probability sum focuses on realistic scenarios, helping optimize production speed or cycle time to cut costs while meeting demand. This approach allows businesses to plan using real-world data, reducing risks from market uncertainty. In summary, the probability sum ensures accurate calculations and boosts inventory management efficiency, offering significant benefits for production managers facing unpredictable demand.

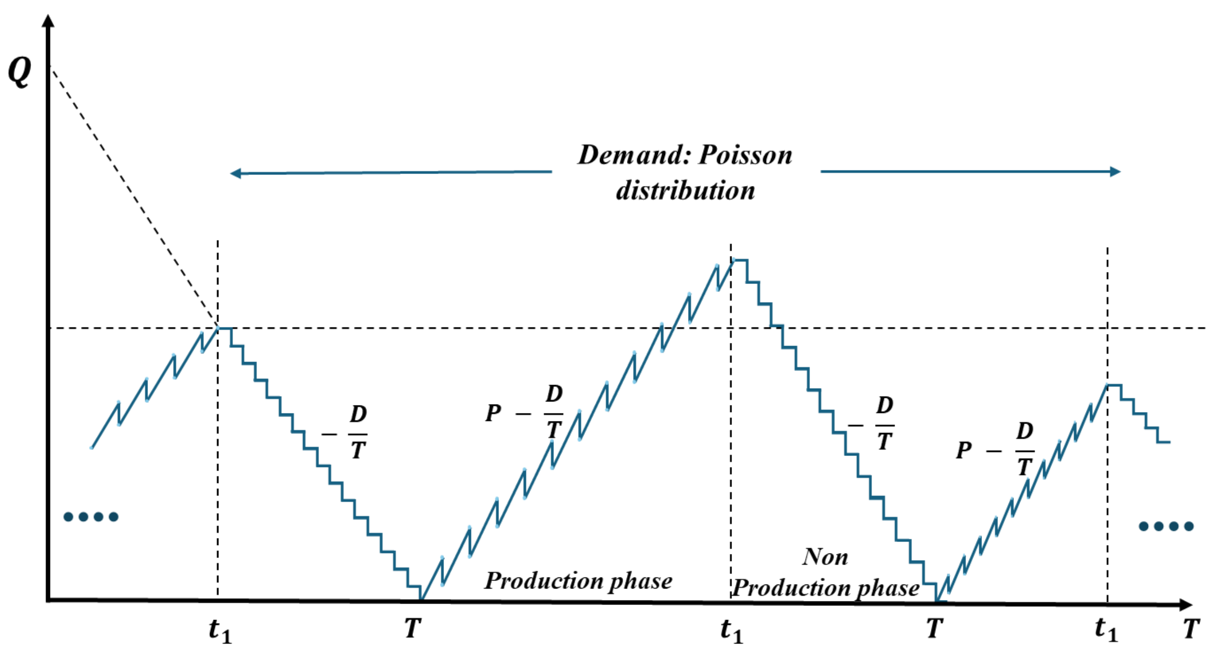


Figure 4.1 Stock level and the corresponding machine statuses in a cycle during a non - shortage case

- The inventory level $I_1(t)$, on hand holding, follows the following differential equations

$$\frac{DI_1(t)}{Dt} = \begin{cases} P - \frac{D}{T} \quad \text{With } I_1(0) = 0;\ 0 \leq t \leq t_1 \\ \frac{-D}{T}\ With\ I_1(\mathrm{T}) \geq 0;\ t_1 \leq t \leq T \end{cases} \tag{14}$$

In a production cycle to avoid shortages, inventory changes in two phases. First, starting with an empty warehouse, the production machine runs continuously, producing goods faster than customers buy them. This causes inventory to grow steadily, like water filling a tank faster than it drains. This phase lasts until the machine stops at a set point, when the warehouse reaches its highest inventory level. On a graph, this phase is shown as an upward-sloping line, with time on the horizontal axis (from cycle start to machine stop) and inventory quantity on the vertical axis. This part is often labeled "machine operating" or marked to show production.

In the second phase, after the machine stops, no new goods are added. Customers keep buying at a steady rate, so inventory decreases, like water draining from a tank with no inflow. This phase runs from the machine's stop to the cycle's end, with enough stock to meet all demand without running out. By the end, inventory is at its lowest but still non-negative. On the graph, this phase is a downward-sloping line from the peak to the final level, with time on the horizontal axis and inventory quantity on the vertical axis. It's often labeled "machine off" or "no production." The graph forms a piecewise line: rising to a peak, then falling, showing how inventory ensures a continuous supply. It also helps analyze holding costs, as storing more goods for longer raises storage and maintenance expenses.

The solutions to equation (14), with the assistance of the initial conditions above, are as follows:

$$I_1(t) = \begin{cases} \left(P - \frac{D}{T}\right) t\ với\ 0 \leq t \leq t_1 \\ Pt_1 - \frac{D}{T} t\ với\ t_1 \leq t \leq T \end{cases} \tag{15}$$

4.1.1 Cost components function

a. Holding cost

In inventory management, holding cost covers expenses for storing goods in a warehouse, including rental fees, preservation costs, insurance, and the opportunity cost of capital tied up in inventory. The non-shortage inventory model ensures enough stock to meet all customer demand, avoiding stockouts. The holding cost formula, based on Sarkar et al. [3], applies to cases where demand follows a Poisson distribution, a probability model for random events like customer orders in a specific period. This model is based on the inventory level $I_1(t)$, described over two phases within a cycle of duration T. Phase 1 spans from time 0 to t_1: during this phase, the production machine operates at a rate P, which exceeds the average demand rate $\frac{D}{T}$, causing the inventory level to increase steadily from an initial level of 0. Phase 2 extends from t_1 to T: the machine stops production, and only demand depletes the stock, leading to a gradual decrease in inventory, though it must remain non-zero by the end of the cycle to avoid shortages. The inventory level is specifically expressed as: from 0 to t_1, $I_1(t) = \left(P - \frac{D}{T}\right)t$, and from t_1 to T, $I_1(t) = \left(Pt_1 - \frac{D}{T}t\right)$. The sum $\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_1(t)dt$ represents the total "inventory-time" over the cycle, corresponding to the area under the inventory curve on a graph.

$$HC = \frac{C_h}{T}\sum_{D=0}^{Q}\left(\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_1(t)dt\right)g(D)$$

$$= \frac{C_h}{T}\sum_{D=0}^{Q}\left(\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_1(t)dt\right)g(D)$$

$$= \frac{C_h}{T}\sum_{D=0}^{t_1P}\left[\int_0^{t_1}\left(P - \frac{D}{T}\right)tdt + \int_{t_1}^{T}\left(Pt_1 - \frac{D}{T}t\right)dt\right]g(D)$$

$$= \frac{C_h}{T}\sum_{D=0}^{t_1P}\left[\left(P - \frac{D}{T}\right)\frac{t^2}{2} + Pt_1(P - t_1) - \frac{D}{T}\left(\frac{T^2 - t^2}{2}\right)\right]g(D)$$

$$= \frac{C_h}{T}\sum_{D=0}^{t_1P}\left(\frac{-t_1{}^2P}{2} + t_1PT - \frac{TD}{2}\right)g(D) \tag{16}$$

b. Energy consumption cost

The energy consumption within the system arises from two distinct processes, specifically during the production phases ($t_0 \rightarrow t_1$) and the non-production phases ($t_1 \rightarrow T$) of the machine.

- During the production

The average energy cost during the production phase is calculated using Specific Energy Consumption (SEC). It accounts for varying customer demand, which is random, like the number of orders overtime. Since demand is random, each demand scenario is weighted by its probability, summing from zero demand to the maximum the machine can produce. The result is the average energy cost during the production phase ($t_0 \rightarrow t_1$)), as shown in formula (17).

$$P_c = \sum_{D=0}^{t_1 P} \left\{ \left[\left(\frac{W}{P} + k \right) E \frac{D}{T} \right] g(D) \right\} \tag{17}$$

- Non-production

The energy cost for keeping the machine active without producing goods is calculated as follows. This cost occurs when the machine is powered on but idle, from when production stops until the cycle ends. To calculate it, we take the machine's standby power consumption (energy used per unit of time) and multiply it by the cost per unit of energy (e.g., price per kilowatt-hour) to get the energy cost per unit of time. Then, we multiply this by the duration duration the machine is idle idle (from production stop to cycle end) and divide by the total cycle duration to find the average cost. Since customer demand varies randomly, from zero to the maximum the machine can produce, we consider all possible demand scenarios, weighted by their probability probabilities. The sum of these scenarios gives the average energy cost for keeping the machine in standby during the non-production phase.

$$I_C = \frac{WE}{T}(T - t_1) \sum_{D=0}^{t_1 P} g(D) = WE\left(1 - \frac{t_1}{T}\right) \sum_{D=0}^{t_1 P} g(D) \tag{18}$$

- Related – warehousing energy consumption cos

The SEC_{ware} function was developed while considering the external temperature factor and the level of warehouse occupancy, The Specific Energy Consumption (SEC) function for warehousing, introduced by Zanoni et al. Zanoni et al. (2013), depends on the storage level. Later, Marchi et al.Marchi et al. (2020) extended it by considering the impact of ambient temperature expressed as follows:

$$EC_{ware} = \frac{E}{T}\int_0^T SEC\big(T_w, I_1(t)\big) I_{1_{max}}\,dt$$
$$= \frac{E}{T}\rho\ \sum_{D=0}^{t_1P}\left\{\int_0^T \left[\alpha I_{1_{max}}^{-\beta} + \delta\left(1-\frac{I_1(t)}{I_{1_{max}}}\right)^{\gamma}\right] I_{1_{max}}\,dt\right\} g(D) \tag{19}$$

In which $\alpha I_{1_{max}}^{-\beta}$ is the basic SEC element of the warehouse when filled. According to the function, the level of SEC is inversely proportional to the occupancy rate, meaning that the SEC value increases as storage decreases. Additionally, the elemnt $\delta\left(1-\frac{I(t)}{I_{1_{max}}}\right)^{\gamma}$ represents the energy consumption value when the warehouse is not fully occupied (δ, γ are coefficients). Thus, it can be asserted that a higher occupancy rate leads to greater energy savings. When the value of I(t) is less than $I_{1_{max}}$ the energy value will gradually increase. Finally, the element ρ is the efficiency coefficient of the two elements (COP). The efficiency coefficient ρ in the model is defined by a formula that compares the cooling performance of the system at two different temperature levels. Specifically, ρ is the ratio of the cooling efficiency at a reference temperature (a standard temperature for the system) to the efficiency at the desired warehouse temperature. The formula relies on the differences between the external temperature outside the warehouse, the reference temperature, and the warehouse temperature. The first part, the reference temperature divided by the difference between the external temperature and the reference temperature, indicates how easily the reference temperature can be achieved compared to the outside environment. The second part, the difference between the external temperature and the warehouse temperature divided by the warehouse temperature, reflects the difficulty of maintaining the low warehouse temperature. When these two parts are multiplied, ρ represents the amount of energy required to sustain the warehouse temperature relative to the reference condition. A higher ρ value occurs when the warehouse needs deep cooling or when the outside temperature is high, resulting in greater energy consumption. This formula aids in calculating the energy cost for warehousing, supporting cost and profit optimization in industries like food or pharmaceuticals, where temperature control is critical. Expressed as follows.

$$\rho = \frac{COP_{Tr}}{COP_{Tw}} = \frac{T_r}{T_{hot} - T_r}\,\frac{T_{hot} - T_W}{T_W} \tag{20}$$

- The maximum inventory level is expressed as follows

$$I_{1_{max}} = \left(P - \frac{D}{T}\right) t_1 \tag{21}$$

The formula $I_{1_{max}} = \left(P - \frac{D}{T}\right) t_1$ represents the maximum inventory level reached in a warehouse during a production cycle when there are no shortages, as described in the model. It calculates the peak amount of stock accumulated at the end of the production period. Here, P is the rate at which items are produced, D is the total demand over the cycle time T, and D/T is the average demand rate. The term $P - \frac{D}{T}$ shows the net rate at which inventory builds up, as production exceeds demand. This net rate is multiplied by t_1, the duration of the production period, to determine the total inventory accumulated. The formula is crucial because it helps determine the storage capacity needed and influences holding costs and energy costs for maintaining the warehouse, especially in industries like food or pharmaceuticals where stable inventory levels are essential to avoid shortages. A higher maximum inventory level means more space and energy are required, impacting the overall cost and profit optimization in the model.

Substituting equations (15) and (21) into equation (19) we obtain the function will be expressed as follows:

$$EC_{ware} = \frac{E}{T}\rho \sum_{D=0}^{t_1 P} \left\{ \int_0^T \left[\alpha I_{1_{max}}^{-\beta} + \delta \left(1 - \frac{I_1(t)}{I_{1_{max}}}\right)^{\gamma} \right] I_{1_{max}}\, dt \right\} g(D)$$

$$= \frac{E}{T} \sum_{D=0}^{t_1 P} \left[\int_0^T \alpha I_{1_{max}}^{-\beta+1} \rho D t + \int_0^T \delta \left(1 - \frac{I_1(t)}{I_{1_{max}}}\right)^{\gamma} \rho I_{1_{max}} dt \right] g(D)$$

$$= \frac{E}{T} \sum_{D=0}^{t_1 P} \left[A(t) + \delta \rho I_{1_{max}} B(t) \right] g(D) \tag{22}$$

Where:

$$A(t) = \int_0^T \alpha I_{1_{max}}^{-\beta+1} \rho dt = \alpha I_{1_{max}}^{-\beta+1} \rho \Big|_0^T = \alpha \rho T \left[\left(P - \frac{D}{P}\right) t_1 \right]^{-\beta+1} \tag{23}$$

$$and\ B(t) = B_1(t) + B_2(t)$$

$$B_1(t) = \left[\int_0^{t_1}\left(1 - \frac{\left(P - \frac{D}{T}\right)t}{I_{1max}}\right)^{\gamma} dt\right] = \left[\int_0^{t_1}\left(1 - \frac{\left(P - \frac{D}{T}\right)t}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma} dt\right]$$

$$= \left[\int_0^{t_1}\left(1 - \frac{t}{t_1}\right)^{\gamma} dt\right] = -\left.\frac{\left(1 - \frac{t}{t_1}\right)^{(\gamma+1)}}{(\gamma + 1)}\right|_0^{t_1} = \frac{t_1}{\gamma + 1} \tag{24}$$

and

$$B_2(t) = \int_{t_1}^{T}\left(1 - \frac{Pt_1 - \frac{D}{T}t}{I_{1max}}\right)^{\gamma} dt = \int_{t_1}^{T}\left(1 - \frac{Pt_1 - \frac{D}{T}t}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma} dt$$

$$= \int_{t_1}^{T}\left(\frac{\frac{D}{T}(t - t_1)}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma} dt = \left(\frac{\frac{D}{T}}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma}\int_{t_I}^{T}(t - t_1)^{\gamma} dt$$

$$= \left(\frac{\frac{D}{T}}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma}\frac{(T - t_1)^{\gamma+1}}{(\gamma + 1)} = \frac{D^{\gamma}(T - t_1)^{(\gamma+1)}}{(\gamma + 1)\left(P - \frac{D}{T}\right)^{\gamma} t_1{}^{\gamma}T^{\gamma}} \tag{25}$$

Substituting equations (23) and (24) and (25) into equation (22), we obtain the function will be expressed as follows:

$$EC_{ware} = \frac{E}{T}\sum_{D=0}^{t_1P}\left\{\alpha\rho T\left[\left(P - \frac{D}{P}\right)t_1\right]^{-\beta+1} + \delta\rho I_{1max}\left[\frac{t_1}{\gamma + 1} + \frac{D^{\gamma}(T - t_1)^{(\gamma+1)}}{(\gamma + 1)\left(P - \frac{D}{T}\right)^{\gamma} t_1{}^{\gamma}T^{\gamma}}\right]\right\} g(D) \tag{26}$$

c. Defective cost

The cost for repairing defective items is based on the product of the unit repair cost and the total expected number of defects.

$$D_c = \frac{C_0}{T}P\sum_{t=0}^{t_1}\alpha_i f_i \sum_{D=0}^{t_1P} g(D) \tag{27}$$

The formula represents the cost of repairing defective items in the model, calculated by multiplying the repair cost per defective item by the total expected number of defects. Here, C_0 is the repair cost per defective item, T is the production cycle duration, and P is the production rate. The term $\alpha_i f_i$ denotes the average defect rate at time point i , where α_i

is the percentage of defective items and f_i is the probability of defects occurring at that point. This result is multiplied by P to determine the number of defects in the production lot, then by $\frac{C_0}{T}$ to obtain the average repair cost per cycle. Notably, the study highlights that the probability of defects $\alpha_i f_i$ increases over time during production due to factors such as worker fatigue, equipment wear, or process errors as the system operates continuously. This means the defect rate is higher at later time points in the production cycle compared to the start, resulting in a growing number of defective items and escalating repair costs. This cost is critical in imperfect production systems, such as electronics or textiles, as it impacts total costs and profit, necessitating optimization to mitigate losses from the rising defect probability over time.

d. Setup cost

In the classical EPQ model, the average total inventory cost consists of two main components, with the setup cost mathematically represented by the equation.

$$S_c = \frac{S}{T}\sum_{D=0}^{t_1 P} g(D) \tag{28}$$

In the classical EPQ model, the average total inventory cost consists of two main parts: setup cost and holding cost. The setup cost covers the expenses of preparing for each production run, such as setting up machines or organizing the production line. It is calculated by taking the fixed cost of each setup, dividing it by the duration of a production cycle, and then multiplying by the probability of random demand, which is based on a specific distribution model that accounts for varying demand.

4.1.2 Objective function

a. Revenue function

The profit function calculates the average profit per production cycle. It takes the selling price per product, subtracts the production cost for good products, and multiplies by the number of good products made. The number of good products depends on production speed, time, and defect rate, which rises over time due to worker fatigue or equipment wear, reducing good products and profit. The result is divided by the cycle duration for average profit, then adjusted by a factor for random market demand, modeled by a distribution. This factor reflects how the market accepts products but isn’t a separate market absorption coefficient. The function is vital in industries like electronics or textiles, balancing production and defect costs to maximize profit under uncertain demand.

$$Revenue = \frac{\left[S_p - \mu t_1 P\right] t_1 P}{T} \sum_{D=0}^{t_1 P} g(D) \tag{29}$$

b. The expected profit of case 1(EEPQ)

Consider the case of no shortages it is the profit minus all costs (including (4), (5), (6), (14), (15), (16)) connected with equations (1) and (8) according to the following equation:

$$\boldsymbol{Pr_1(t_1, P)} = \frac{\left[S_p - \mu t_1 P\right] t_1 P}{T} \sum_{D=0}^{t_1 P} g(D) - HC - P_c - I_C - EC_{ware} - D_c - S_c$$

$$= \frac{\left[S_p - \mu t_1 P\right] t_1 P}{T} \sum_{D=0}^{t_1 P} g(D) - \frac{C_h}{T} \sum_{D=0}^{t_1 P} \left(\frac{-{t_1}^2 P}{2} + t_1 PT - \frac{TD}{2}\right) g(D)$$

$$- \sum_{D=0}^{t_1 P} \left\{\left[\left(\frac{W}{P} + k\right) E \frac{D}{T}\right] g(D)\right\} - WE\left(1 - \frac{t_1}{T}\right) \sum_{D=0}^{t_1 P} g(D)$$

$$- \frac{E}{T} \sum_{D=0}^{t_1 P} \left[\alpha \rho T \left[\left(P - \frac{D}{T}\right) t_1\right]^{-\beta+1} + \delta \rho \left(P - \frac{D}{T}\right) t_1 \left[\frac{t_1}{\gamma + 1} + \frac{D^\gamma (T - t_1)^{(\gamma+1)}}{(\gamma + 1)\left(P - \frac{D}{T}\right)^\gamma {t_1}^\gamma T^\gamma}\right]\right] g(D$$

$$- \frac{C_0}{T} P \sum_{t=0}^{t_1} \alpha_i f_i \sum_{D=0}^{t_1 P} g(D) - \frac{S}{T} \sum_{D=0}^{t_1 P} g(D) \tag{30}$$

The objective function in the EEPQ model is designed to help production managers optimize profit by balancing revenue and associated costs. Specifically, profit is calculated by taking the expected total revenue from product sales and subtracting various costs, including inventory holding costs, energy costs during production, energy costs when machines are idle, energy costs for warehouse maintenance, costs for handling defective products, and setup costs for each production cycle. The expected revenue is based on the selling price of the products, the quantity of goods produced within a certain time at a specific rate, and the ability to meet varying market demand levels, ranging from no demand to the maximum the system can supply. By considering all possible demand scenarios, the objective function ensures that decisions regarding production time and rate are based on realistic data, minimizing risks from market fluctuations. Its significance lies in providing a powerful tool for managers to plan production efficiently, optimize costs, and enhance the ability to meet demand, thereby improving profitability and inventory management efficiency in an imperfect production environment.

4.2 Scenario Shortages (EEPQ_S)

The EEPQ_S model, an upgrade of EEPQ, allows shortages when demand exceeds stock in a production cycle. In retail, logistics, or custom manufacturing, customers may wait for unavailable goods, but this causes costs like late delivery penalties, backorder management, or loss of trust. Designed for imperfect production with defective products and random demand, EEPQ_S calculates production based on time and speed, setting a supply limit. It uses a probability sum to evaluate all demand levels, from zero to beyond capacity, modeling demand as random events. Unlike EEPQ, which avoids shortages with high inventory, EEPQ_S includes demand exceeding supply, factoring in shortage costs. This assesses if allowing shortages is more efficient than high inventory costs. The probability sum calculates production($\sum_{D=t_1P+1}^{\infty}(cost\ function)g(D)$), inventory, and shortage costs, aiding efficiency comparisons. Ideal for volatile markets, it helps managers weigh lower inventory costs against shortage expenses, optimizing production strategies .

This enables managers to determine whether reducing production and accepting occasional shortages can lower overall costs. For instance, during low-demand periods, the EEPQ_S model can help minimize excess inventory, while still managing shortage costs during peak periods. The probability sum supports the optimization of decisions, such as setting production speed and cycle time to balance production, holding, and shortage costs. In conclusion, the probability sum in the EEPQ_S model enhances its practical relevance by providing a robust framework for comparing the efficiency of shortage and no-shortage scenarios, empowering businesses to make cost-effective and flexible decisions in volatile market conditions. The inventory level $I_2(t)$, on hand holding, follows the following differential equations:

$$\frac{DI(t)}{Dt} = \begin{cases} P - \dfrac{D}{T} \text{ with } I_2(0) = 0 \; and \; I_2(T) = 0 \\ (0 \le t \le t_1 \; and \; t_3 \le t \le T) \\ \dfrac{D}{T} \text{ with } I_2(0) = 0 \; and \; I_2(t_3) \le 0 \\ (0 \le t \le t_1 \; and \; t_2 \le t \le t_3) \end{cases} \quad (31)$$

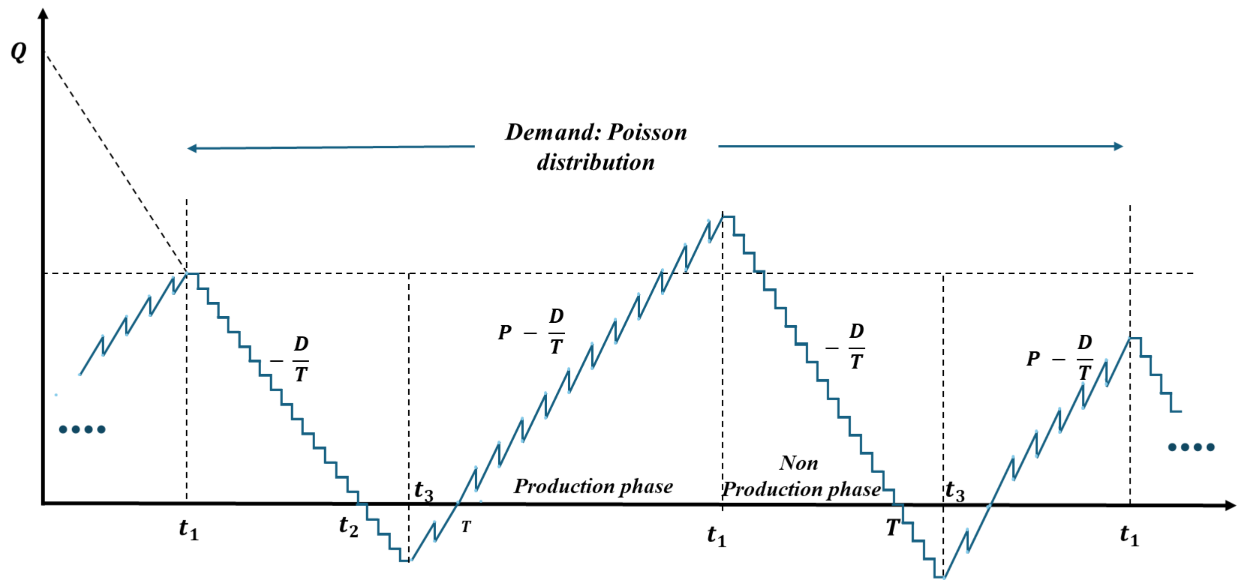


Figure 4.2: Stock level and the corresponding machine statuses in a cycle during a shortage case

The solutions to equation (31), with the assistance of the initial conditions above, are as follows:

$$I_2(t) = \begin{cases} \left(P - \dfrac{D}{T}\right) t \; với \; 0 \le t \le t_1; \; t_3 \le t \le T \\ Pt_1 - \dfrac{D}{T} t \; với \; t_1 \le t \le t_2 \\ \dfrac{-D}{T}(t - t_2) \; với \; t_2 \le t \le t_3 \end{cases} \quad (32)$$

Where:

$$Q = Pt_1 < D; \quad (33)$$

$$Pt_1 = \frac{D}{T} t_2 \rightarrow Pt_1 \frac{T}{D} = t_2 \quad (34)$$

$$(T - t_3)P + Pt_1 = D \rightarrow TP - t_3 P + Pt_1 = D \quad (35)$$

$$P(T + t_1) - D = t_3 \rightarrow Pt_3 = (T + t_1) - \frac{D}{P} \quad (36)$$

4.2.1 Cost components function

a. Inventory cost

According to Sarkar et al. [29] the holding costs for items in shortages inventory models with demand distributed according to the Poisson distribution are formulated as follows

$$HC^2 = \frac{C_h}{T}\sum_{D=Q+1}^{\infty}\left(\int_0^{t_1} I_2(t)dt + \int_{t_1}^{t_2} I_2(t)dt\right)g(D)$$

$$= \frac{C_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\int_0^{t_1}\left(P - \frac{D}{T}\right)tdt + \int_{t_1}^{t_2}\left(Pt_1 - \frac{D}{T}t\right)dt\right]g(D)$$

$$= \frac{C_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\frac{Q^2}{2P}\left(\frac{PT}{D} - 1\right)\right]g(D) = \frac{C_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\frac{{t_1}^2P}{2}\left(\frac{PT}{D} - 1\right)\right]g(D) \tag{37}$$

b. Energy consumption cost

The energy consumption within the system arises from two distinct processes, specifically during the production phases ($t_0 \rightarrow t_1$ and $t_3 \rightarrow t_4$) and the non-production phases ($t_1 \rightarrow t_3$) of the machine.

- During production

By using SEC, the average energy consumtion cost during production time ($t_0 \rightarrow t_1$ and $t_3 \rightarrow t_4$) is formulated as (23).

$$P_c^{\ 2} = \sum_{D=t_1P+1}^{\infty}\left(\frac{W}{P} + k\right)E\frac{D}{T}g(D) \tag{38}$$

- Non-production

The energy cost incurred to keep the machine active without performing any operational tasks is defined by equation

$$I_c{}^2 = \sum_{D=t_1P+1}^{\infty} \frac{WE}{T}(t_3 - t_1)g(D) = \sum_{D=t_1P+1}^{\infty} WE\left(1 - \frac{D}{PT}\right)g(D) \tag{39}$$

- **Related – warehousing energy consumption cost**

The SEC_{ware} function was developed while considering the external temperature factor and the level of warehouse occupancy, expressed as follows:

$$EC_{ware}{}^2 = \frac{1}{T}E\int_0^T SEC\big(T_w, I_2(t)\big)I_{2_{max}}\,dt$$

$$= \frac{E}{T}\rho\sum_{D=t_1P+1}^{\infty}\left\{\int_0^T\left[\alpha I_{2_{max}}^{-\beta} + \delta\left(1 - \frac{I_2(t)}{\rho I_{2_{max}}}\right)^{\gamma}\right]I_{2_{max}}\,dt\right\}g(D) \tag{40}$$

In which $\alpha I_{1_{max}}^{-\beta}$ is the basic SEC element of the warehouse when filled. According to the function, the level of SEC is inversely proportional to the occupancy rate, meaning that the SEC value increases as storage decreases. Additionally, the elemnt $\delta\left(1 - \frac{I(t)}{I_{1_{max}}}\right)^{\gamma}$ represents the energy consumption value when the warehouse is not fully occupied (δ, γ are coefficients). Thus, it can be asserted that a higher occupancy rate leads to greater energy savings. When the value of I(t) is less than $I_{1_{max}}$ the energy value will gradually increase.

- The maximum inventory level is expressed as follows:

$$I_{2_{max}} = I_2(t) = \left(P - \frac{D}{T}\right)t_1 \tag{41}$$

- Substituting equations (32); (34); (36) and (41) into equation (40), we obtain the function will be expressed as follows:

$$TC_{ware}{}^2 = \frac{E}{T}\sum_{D=t_1P+1}^{\infty}\big[A(t) + \delta\rho I_{2_{max}}B(t)\big]g(D) \tag{42}$$

Where:

$$A(t) = \int_0^T \alpha I_{2_{max}}^{-\beta+1}\rho\,dt = \alpha I_{2_{max}}^{-\beta+1}\rho\Big|_0^T = \alpha\rho T\left[\left(P - \frac{D}{P}\right)t_1\right]^{-\beta+1} \tag{43}$$

$$B(t) = B_1(t) + B_2(t) + B_{3,4(t)}$$

$$B_1(t) = \left[\int_0^{t_1} \left(1 - \frac{\left(P - \frac{D}{T}\right)t}{I_{2max}}\right)^{\gamma} dt\right] = \left[\int_0^{t_1} \left(1 - \frac{\left(P - \frac{D}{T}\right)t}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma} dt\right]$$

$$= \left[\int_0^{t_1} \left(1 - \frac{t}{t_1}\right)^{\gamma} dt\right] = -\frac{\left(1 - \frac{t}{t_1}\right)^{(\gamma+1)}}{(\gamma+1)}\Bigg|_0^{t_1} = \frac{t_1}{\gamma+1} \tag{44}$$

$$B_2(t) = \left[\int_{t_1}^{t_2} \left(1 - \frac{Pt_1 - \frac{D}{T}t}{I_{2max}}\right)^{\gamma} dt\right] = \left[\int_{t_1}^{t_2} \left(\frac{\frac{D}{T}(t - t_1)}{\left(P - \frac{D}{T}\right)t_1}\right)^{\gamma} dt\right]$$

$$= \frac{D^{\gamma}(t_2 - t_1)^{(\gamma+1)}}{(\gamma+1)\left(P - \frac{D}{T}\right)^{\gamma} t_1{}^{\gamma} T^{\gamma}} = \frac{D^{\gamma} t_1{}^{(\gamma+1)} \left(\frac{PT - D}{D}\right)^{(\gamma+1)}}{(\gamma+1)\left(P - \frac{D}{T}\right)^{\gamma} t_1{}^{\gamma} T^{\gamma}}$$

$$= \frac{t_1(PT - D)}{D(\gamma+1)} \tag{45}$$

$$B_{3,4}(t) = \left[\int_{t_2}^{T} \left(1 - \frac{0}{I_{2max}}\right)^{\gamma} dt\right] = T\left(1 - \frac{Pt_1}{D}\right) \tag{46}$$

Substituting equations (43); (44); (45); and (46) into equation (42), we obtain the function will be expressed as follows:$EC_{ware}{}^{2}$

$$= \frac{E}{T} \sum_{D=t_1P+1}^{\infty} \left[\alpha\rho T\left[\left(P - \frac{D}{P}\right)t_1\right]^{-\beta+1} + \delta\rho I_{2max}\left[\frac{t_1}{\gamma+1} + \frac{t_1(PT - D)}{D(\gamma+1)} + T\left(1 - \frac{Pt_1}{D}\right)\right]\right] g(D) \tag{47}$$

c. Shortage cost

The shortage cost from phases (t_2 → t_3)is a complete stockout, with no goods available to meet demand. Meanwhile, from (t_3 → t_4), the shortage occurs while production has resumed, but the supply of goods is still insufficient. The shortage cost in the EEPQ_S model is calculated when there isn't enough stock in the warehouse to meet demand, occurring in two phases: from when the warehouse runs out of stock until production restarts, and from when production resumes but the supply is still insufficient. This cost takes the cost per unit of missing stock, divides it by the production cycle duration, and multiplies it by the total expected shortage when demand exceeds the produced stock. In the first phase, when the warehouse is completely empty, the shortage grows based on the average demand rate. In the second phase, when production has restarted but isn't fast enough to meet demand, the shortage is based on the gap between the production speed and demand. The result is adjusted for the likelihood of different random demand levels, using a specific distribution model.

$$Sh_c{}^2 = \frac{S_h}{T}\sum_{D=t_1P+1}^{\infty}\left\{\left[\int_{t_2}^{t_3}\frac{D}{T}(t-t_2)\,dt\right]+\left[\int_{t_3}^{T}\left(P-\frac{D}{T}\right)(t-t_3)dt\right]\right\}g(D)$$

$$= \frac{S_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\frac{D}{T}\frac{(t-t_2)^2}{2}\Big|_{t_2}^{t_3}+\left(P-\frac{D}{T}\right)\frac{(t-t_3)^2}{2}\Big|_{t_3}^{T}\right]g(D)$$

$$= \frac{S_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\frac{D}{T}\frac{(t_3-t_2)^2}{2}+\left(P-\frac{D}{T}\right)\frac{(T-t_3)^2}{2}\right]g(D)$$

$$= \frac{S_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\frac{D}{T}\frac{\left[(T+t_1)-\frac{D}{P}-Pt_1\frac{T}{D}\right]^2}{2}+\left(P-\frac{D}{T}\right)\frac{\left[(T-(T+t_1)+\frac{D}{P})\right]^2}{2}\right]g(D)$$

$$= \frac{S_h}{T}\sum_{D=t_1P+1}^{\infty}\left[\frac{D}{2T}\left(T+t_1-\frac{D}{P}-\frac{Pt_1T}{D}\right)^2+\frac{1}{2}\left(P-\frac{D}{T}\right)\left(\frac{D}{P}-t_1\right)^2\right]g(D) \tag{48}$$

d. Defective cost

The cost for repairing defective items is based on the product of the unit repair cost and the total expected number of defects.

$$D_c{}^2 = \frac{C_0}{T}\left(P\sum_{t=0}^{t_1}\alpha_i f_i + P\sum_{t=0}^{T-t_3}\alpha_i f_i\right)\sum_{D=t_1P+1}^{\infty}g(D) \tag{49}$$

e. Setup cost

In the classical EPQ model, the average total inventory cost consists of two main components, with the setup cost mathematically represented by the equation.

$$S_c{}^2 = \frac{S}{T}\sum_{D=t_1P+1}^{\infty} g(D) \tag{50}$$

4.2.2 Objective function

a. Revenue function

$$Revenue = \frac{\left[S_p - \mu(t_1 + T - t_3)P\right](t_1 + T - t_3)P}{T}\sum_{D=0}^{t_1P} g(D) \tag{51}$$

The revenue function in the EEPQ model is designed to calculate expected profit, assisting production managers in optimizing economic efficiency. Profit is determined based on the expected revenue from product sales, calculated from the maximum selling price of the products and the quantity of goods produced, while considering the influence of the market absorption coefficient, which reflects the market's acceptance of the product. The quantity of products is produced in two phases: the initial production phase and an additional production phase after a machine downtime, ensuring sufficient supply to meet market demand. The output depends on the production rate and the total production time across both phases. The expected revenue is adjusted based on the ability to meet various market demand levels, ranging from no demand to the maximum the system can supply. By considering all possible demand scenarios, the revenue function ensures that decisions regarding production time and rate are based on realistic data, helping to mitigate risks from market fluctuations. Its significance lies in providing an effective tool for managers to plan production accurately, optimize revenue by balancing output and demand, and thereby enhance profitability and management efficiency in an imperfect production environment.

b. The expected profit of case 2(EEPQ_S):

Consider the case of shortages it is the profit minus all costs (including (22); (23); (24); (32); (33); (34); (35)) connected with equations (1) and (8) according to the average total profit following equation:

$$\boldsymbol{Pr_2(t_1, P)} = \frac{\left[S_p - \mu(t_1 + t_4 - t_3)P\right](t_1 + t_4 - t_3)P}{T}\sum_{D=0}^{t_1P} g(D)$$

$$-HC^2 - {P_c}^2 - {I_c}^2 - {EC_{ware}}^2 - {Sh_c}^2 - {D_c}^2 - {S_c}^2$$

$$= \frac{\left[S_p - \mu(t_1 + T - t_3)P\right](t_1 + T - t_3)P}{T} \sum_{D=0}^{t_1 P} g(D)$$

$$-\frac{C_h}{T} \sum_{D=t_1P+1}^{\infty} \left\{\left[\frac{{t_1}^2 P}{2}\left(\frac{PT}{D} - 1\right)\right] g(D)\right\} - \sum_{D=t_1P+1}^{\infty} \left[\left(\frac{W}{P} + k\right) E \frac{D}{T} g(D)\right]$$

$$- \sum_{D=t_1P+1}^{\infty} \left[WE\left(1 - \frac{D}{PT}\right) g(D)\right]$$

$$-\frac{E}{T}\langle \sum_{D=t_1P+1}^{\infty} \left[\alpha\rho T\left\{\left[\left(P - \frac{D}{P}\right)t_1\right]\right\}^{-\beta+1} + \delta\rho\left(P - \frac{D}{T}\right)t_1\left[\frac{t_1}{\gamma+1} + \frac{t_1(PT - D)}{D(\gamma+1)} + T\left(1 - \frac{Pt_1}{D}\right)\right]\right] g(D)\rangle$$

$$- \frac{S_h}{T} \sum_{D=t_1P+1}^{\infty} \left[\frac{D}{2T}\left(T + t_1 - \frac{D}{P} - \frac{Pt_1T}{D}\right)^2 + \frac{1}{2}\left(P - \frac{D}{T}\right)\left(\frac{D}{P} - t_1\right)^2\right] g(D)$$

$$-\frac{C_0}{T}\left(P\sum_{t=0}^{t_1} \alpha_i f_i + P\sum_{t=0}^{t_4 - t_3} \alpha_i f_i\right) \sum_{D=t_1P+1}^{\infty} g(D) - \frac{S}{T}\sum_{D=t_1P+1}^{\infty} g(D) \tag{52}$$

The expected profit in case 2 (EEPQ_S) of the EEPQ model is designed to calculate profit in the context of shortages, assisting production managers in optimizing financial performance. Specifically, the average profit is determined by taking the expected revenue from product sales and subtracting all related costs, including inventory holding costs, energy costs during production, energy costs when machines are idle, energy costs for warehouse maintenance, costs incurred due to shortages, costs for handling defective products, and setup costs for each production cycle. The expected revenue is calculated based on the selling price of the products, the quantity of goods produced within a specific time at a defined rate, and the ability to meet market demand levels ranging from no demand to the maximum the system can supply. The costs are computed based on equations related to production time, additional time, and waiting time, reflecting the complexity of the system when shortages occur. By considering all possible demand scenarios, this profit function ensures that decisions regarding production time and rate are based on realistic data, mitigating risks from market fluctuations and shortages.

CHAPTER 5: RESOLUTION APPROACH AND NUMERICAL ANALYSIS

Chapter 5 builds on the models developed in Chapter 4, focusing on the resolution approach and numerical analysis to evaluate the effectiveness of the EEPQ models. This chapter outlines the solution methods, including algorithms for solving multi-objective problems, to optimize decision-making in production management. Additionally, it provides specific numerical examples and sensitivity analysis to assess the robustness and practical applicability of the models. This content plays a critical role in validating and refining production management strategies.

5.1 Resolution approach

5.1.1 Solution method

The production optimization problem is designed to maximize the average profit function.

$$Maximum\ \ Pr_i(t_1, P\ )$$

By determining the optimal values for production rate (P) and production time (t_1), subject to the constraint $P_{min}\ \le\ P\ \le\ P_{max}$. However, this objective function exhibits significant complexity due to the integration of stochastic elements, such as the Poisson distribution, and multiple nonlinear terms. Specifically, in the case of shortages (EEPQ_S), the profit equation (36) encompasses revenue from sales and production, minus intricate costs including holding costs, production costs, warehousing costs, shortage costs, distribution costs, and setup costs. The presence of infinite sums and nonlinear components in this equation renders traditional analytical methods or exact numerical techniques infeasible, necessitating an alternative approach to obtain optimal solutions.

Given the complexity of the objective function, the use of commercial solvers such as LINGO or MATLAB was considered, but these tools are not guaranteed to be effective for handling nonlinear problems with stochastic components like this one. Instead, three numerical optimization algorithms were proposed and employed: Differential Evolution (DE), Genetic Algorithm (GA), and Grid Search. DE is a powerful evolutionary algorithm that optimizes populations of solutions, making it well-suited for non-differentiable and nonlinear problems. GA mimics natural selection processes, using mechanisms like crossover and mutation to search for global optima, proving effective in complex parameter spaces. GS, though simpler, systematically evaluates all parameter combinations within a defined grid, ensuring accuracy in problems with constrained search spaces. These algorithms were implemented using Python, leveraging libraries such as NumPy for matrix computations, SciPy for advanced optimization tools, Matplotlib for visualizing the profit surface, thereby facilitating the analysis and identification of optimal values t_1* and P*.

The adoption of these numerical methods not only overcomes the challenges posed by the stochastic and nonlinear nature of the objective function but also delivers reliable results and clear visualizations. The DE and GA algorithms enable flexible exploration of the solution space, avoiding local optima traps, while Grid Search provides a comprehensive approach to ensure no potential solutions are overlooked. Support from Python libraries accelerates computations and generates insightful visualizations, aiding managers in making informed decisions about production parameters. Consequently, this approach successfully addresses the production optimization problem and establishes a robust framework for tackling similar complex problems in the future.

5.1.2 Algorithm for solving multi-objective problems

a. Choosing a solver

Choosing the right algorithm is key to solving optimization problems efficiently and accurately. This study compares three methods: Differential Evolution (DE), Genetic Algorithm (GA), and Grid Search (GS). Each has unique features suited for complex problems, like optimizing an average profit function with random and nonlinear factors. The algorithm choice affects convergence speed and the ability to find the global optimum. DE handles non-differentiable functions well, GA explores wide search spaces flexibly, and GS ensures reliable results within a limited range. DE uses vector mutation for fast convergence on nonlinear problems, GA mimics natural evolution to avoid local optima, and GS evaluates all points in a grid for precision in a defined space. Each has strengths: DE for speed, GA for flexibility, and GS for accuracy in small ranges. However, selecting the best method requires analyzing their drawbacks and the problem's specific context.

This study evaluates three optimization algorithms: Differential Evolution (DE), Genetic Algorithm (GA), and Grid Search (GS), each with unique strengths for different optimization problems. DE, an evolutionary algorithm, excels in nonlinear and non-differentiable problems. It uses vector differences for mutation, exploring the search space flexibly without needing derivatives, converging quickly, and avoiding local optima. DE is easy to configure with few parameters (mutation and crossover rates), making it practical for real-world use. GA, another evolutionary algorithm, mimics natural evolution through selection, crossover, and mutation. It's versatile for both continuous and discrete problems, doesn't require differentiable functions, and explores wide search spaces to avoid local optima. GA also supports parallel computing, speeding up large-scale problems. GS, a simple method, suits problems with small parameter spaces. It evaluates every point in a predefined grid, ensuring the best solution within that range. A dense grid allows a thorough global search, and GS's lack of randomness ensures consistent, reproducible results, ideal for transparent scientific research. Despite their strengths, DE, GA, and GS have limitations to consider before use. DE's main weakness is its sensitivity to parameter tuning. Poor choices for mutation rate or population size can slow convergence or trap it in local optima. DE also struggles with complex constraints, needing extra techniques like penalty functions, which complicates implementation. GA's biggest drawback is high computational cost, as it maintains large populations over many generations, slowing down for large search spaces or complex functions. GA doesn't guarantee global optima, often giving near-optimal solutions, which may not suit high-precision needs. Designing effective crossover and mutation operators requires expertise, as poor designs harm performance. GS's key limitation is its computational cost, which skyrockets in large or high-resolution search spaces, especially in multi-dimensional problems where points grow exponentially. GS relies on grid design; a poorly designed or coarse grid may miss optimal solutions. GS works best for small, constrained parameter spaces, otherwise wasting resources.

After thorough analysis, Grid Search (GS) is chosen as the most appropriate optimization method for this problem. The main reason is that GS proves effective in contexts where the search space is limited, for example when parameters such as production time (t_1) and production rate (P) lie only within the defined interval $[P_{min} \leq P \leq P_{max}]$. In this case, the computational cost of GS can be controlled through reasonable grid design, while ensuring that the best solution within the grid range is found. In particular, if the grid is designed densely enough, GS has the ability to find the global optimum something that DE and GA cannot guarantee due to their heuristic nature, which is susceptible to randomness. Another important reason for choosing GS is its superior transparency and reproducibility. Unlike DE and GA which are affected by randomness or parameter tuning, GS does not rely on stochastic elements, ensuring consistent and verifiable results. This is crucial in scientific research, where reliability and reproducibility are core requirements. Although GS's computational cost can rise when the grid is large, this drawback is easily mitigated in a limited search space with modern computing resources. GS provides a clear, easy-to-understand method that does not require deep expertise in optimization, while offering a balance of simplicity, reliability, and the ability to locate optimal solutions. Compared to DE and GA, GS excels at meeting the demands for transparency and reproducibility, especially when thorough analysis of optimal parameters within a constrained space is required. Moreover, this problem has a complex objective function, including random components (such as a Poisson distribution) and nonlinear factors, increasing the risk that heuristic algorithms like DE and GA become trapped in local optima. GS, with its ability to comprehensively evaluate across the grid, effectively mitigates this risk. In addition, GS is easy to implement thanks to support from Python libraries such as NumPy, SciPy, and Matplotlib, allowing clear visualization of results and analysis of the profit surface. Compared with DE and GA, GS offers greater simplicity, higher reliability, and the ability to precisely determine the optimal values t_1* and P* in this specific context, where transparency and reproducibility play a crucial role in evaluating and implementing the solution.

In conclusion, Grid Search (GS) is the optimal choice for the average profit optimization problem in this study, thanks to its ability to conduct precise searches within a limited parameter space, its transparency, and the simplicity of its implementation. While DE and GA may be more suitable for problems with large search spaces or requiring global exploration, GS proves superior in this specific context, where accuracy and reproducibility are prioritized. However, to enhance effectiveness in the future, one could consider combining GS with other methods for example, using DE or GA to narrow the search space first, then applying GS to fine-tune the results. This approach could leverage the strengths of all three algorithms, opening new directions for tackling more complex optimization problems.

b. The process of developing an optimization algorithm in Python

The process of solving the profit optimization problem in a production system is a complex yet systematic endeavor, utilizing Grid Search (GS) as the primary tool to identify the optimal solution. The goal is to determine the optimal values for decision variables, such as production time and production rate, to maximize profit while adhering to practical constraints and addressing uncertainties like fluctuating market demand. Below is a detailed breakdown of each step in the process.

- Constructing the Mathematical Model. The mathematical model is established to capture the key factors influencing profit in the production system. The main components include: Revenue, Inventory Cost., Shortage Cost, Energy Cost, Defective Product Cos, Setup Cost,.
- Designing the Profit Function
    - The profit function is developed to evaluate the economic performance of the system, defined as revenue minus total costs. To handle randomness in market demand, the function employs the following techniques:
    - Demand Range: A finite range of demand values is selected from the Poisson distribution to ensure computational feasibility.
    - Probabilities: Each demand value is assigned a probability weight based on the Poisson probability density function.
    - Vectorization: Computations are performed simultaneously on numerical arrays (using libraries like NumPy), enhancing processing efficiency.

Additionally, the profit function is designed to handle exceptions, such as division by zero, by returning a negative infinity value (-∞). This ensures stability and prevents errors during calculations.

- Optimization Using Grid Search

Grid Search is the core method for identifying the optimal combination of decision variables, such as production time (t_1) and production rate (P). This method operates systematically and exhaustively, ensuring a thorough exploration of the solution space. Below is a detailed analysis of the steps involved:

- Defining the Parameter Grid

A parameter grid is created for each decision variable. For example:

Production time (t_1) may range from 1 to an upper limit (e.g., 30), with a step size of 1 unit.

Production rate (P) is evaluated within a range of 150 to 300, also with a step size of 1. This grid defines all possible combinations of variable values, forming a parameter space for exploration.

- Evaluating Each Grid Point

For each combination of values (t_1 , P) in the grid, the profit function is called to compute the corresponding profit value. This process is conducted sequentially and comprehensively, ensuring no combination is overlooked. Practical constraints, such as t_1 not exceeding the cycle time (t_c) or P remaining within a valid range, are also enforced during this step.

- Tracking and Updating the Best Result

Throughout the evaluation, the system tracks the combination of variables yielding the highest profit. Invalid values, such as NaN or infinity (±∞), are discarded to ensure the final result is accurate and meaningful. A temporary variable stores the maximum profit and its corresponding combination, updated continuously as better solutions are found.

- Displaying Progress

To track progress in large parameter spaces, the system shows updates after every 1000 points evaluated, including points processed, total points, and completion percentage. This helps estimate remaining time and confirms the process is on track. Grid Search (GS) ensures the global optimum with a fine grid but can be slow in high-dimensional or large spaces due to many combinations. Still, GS is reliable in this problem for its transparency and thorough exploration. After finding the optimal solution, an analysis function breaks down profit components (revenue, inventory cost, energy cost, etc.) into a dictionary, allowing users to evaluate each factor's impact. It handles errors to ensure valid results for decision-making. The profit optimization uses GS with mathematical modeling and modern computing. From building a complex objective function to using GS for optimization, analyzing profit, and visualizing results, this process ensures accuracy and supports effective decision-making in production systems. GS's transparency and systematic approach make it ideal for reliable optimization.

5.2 Numerical Analysis

This study focuses on conducting a detailed quantitative analysis to elucidate the characteristics and efficacy of the EEPQ_S model, an advanced production inventory model that integrates energy considerations. Through a systematic analytical process, encompassing case-specific resolution, energy impact analysis, and sensitivity analysis, the study not only evaluates the model's applicability in real-world production systems but also highlights its advantages over the traditional EPQ_S model.

The study first applied the EEPQ_S model to a real-world case using research data to test its effectiveness in imperfect production systems with potential failures, uncertain demand, and possible shortages. By simulating realistic scenarios, it showed how the EEPQ_S model adjusts production decisions like lot size and inventory to optimize performance. Results showed the model adapts well to production fluctuations. For example, in a high-failure scenario, it adjusted the production cycle to reduce waste while ensuring enough inventory for volatile demand, making it a reliable tool for managers facing market uncertainties. Next, the study evaluated energy costs in the EEPQ_S model compared to the traditional EPQ_S model, which ignores energy costs. It analyzed how EEPQ_S balances energy, production, and shortage costs in imperfect systems. Two variants were compared: one allowing shortages and one prohibiting them. With shortages allowed, EEPQ_S performed better by factoring in energy costs, reducing the financial impact of disruptions like shortages. For instance, during a demand surge, it adjusted production frequency to save energy. Without shortages, it optimized cycles and safety stock, maintaining stability despite high failure rates. These findings highlight EEPQ_S's flexibility and the importance of including energy costs in production management, especially in systems with demand volatility and technical errors.

Table 5.1: Differences Between Models

MODELS	FAILURE RATE	SHORTAGE	ENERGY IMPLICATION
EPQ-S	✓	✓	
EEPQ	✓		✓
EEPQ_S	✓	✓	✓

Finally, the study performed a sensitivity analysis to evaluate the impact of input parameters such as energy costs, production failure rates, shortage costs, and market demand on the performance of the EEPQ_S model. This process involved varying each parameter within a defined range and observing its effects on key factors, including lot size, production cycle, and inventory costs. The results revealed that energy costs and production failure rates significantly influence the model's performance. For instance, a 20% increase in energy costs prompted the EEPQ_S model to adopt smaller lot sizes to reduce energy consumption while still meeting demand. Based on these findings, the study proposed specific optimization strategies, such as enhancing inventory management or adjusting production frequency, to improve the model's efficiency in real-world scenarios. This study provides a comprehensive analysis of the EEPQ_S model, clarifying its applicability in imperfect production systems characterized by uncertain demand and potential shortages. Compared to the traditional EPQ_S model, the EEPQ_S model demonstrates superior performance by integrating energy costs and effectively managing factors such as production failures and inventory shortages. The results from the sensitivity analysis further reinforce the model's practicality and offer actionable recommendations for optimization, establishing the EEPQ_S model as a valuable tool for production managers seeking to make informed decisions in the complex landscape of modern manufacturing.

5.2.1 Numerical examples

To ensure the robustness and practical applicability of the EEPQ_S model, the study employed a comprehensive set of input parameters sourced from reputable scientific literature. These parameters were categorized into three primary groups: manufacturing process parameters, warehousing parameters, and shortage-related parameters. They were sourced from the studies by H.-N. Nguyen et al. [[19], [42]], providing a reliable dataset for simulating real-world production conditions. The specific parameters and their sources are detailed as follows:

Table 5.2: Summarizes the data set used in all the examples presented in this section.

λ	3000	$unit/h$	S_p	2.5	$\$/unit$
S	500	$\$/h$	T	30	h
H	$1 \cdot 10^{-3}$	$\$/(unit \cdot h)$	α	$445 \cdot 10^{-4}$	
W	100	kW	μ	0.0005	
K	0.05	$kWh/unit$	δ	$171.2 \cdot 10^{-4}$	
E	0.2	$\$/(kWh)$	γ	0.5	
S_h	0.04	$\$/(unit \cdot h)$	T_w	-18	$°C$
C_0	1	$\$/(unit \cdot h)$	T_r	6	$°C$
P	$[150{:}\,300]$	$unit/h$	T_{hot}	16	$°C$
β	*0.23*				

- **Manufacturing Process Parameters**: This group includes the demand rate (λ), setup cost (*S*), holding cost (*H*), minimum production rate (P_{min}), maximum production rate (P_{max}), unit production cost (*W*), fixed cost per production cycle (*K*), and energy cost per unit (*E*). These parameters define the operational constraints and cost structure of the production system, enabling the EEPQ_S model to accurately reflect variations in production efficiency and energy consumption.

- **Warehousing Parameters**: This group comprises the deterioration rate (α), mean demand rate (μ), demand variability (δ), and a scaling factor for inventory holding costs (γ). These parameters capture the dynamics of inventory management in systems affected by product deterioration and unstable demand. By incorporating these parameters, the EEPQ_S model effectively addresses the challenges of maintaining optimal inventory levels under varying storage conditions, thereby enhancing inventory management efficiency.

- **Shortage-Related Parameters**: This group includes the shortage cost per unit (C_0) and the cost of maintaining safety stock (C_h). These parameters reflect the financial implications of inventory shortages and the strategic decisions required to mitigate their impact. The EEPQ_S model leverages these parameters to balance the trade-offs between allowing shortages and maintaining excess inventory, optimizing total costs across diverse production scenarios.

- **Defective Item Parameter**: This parameter, representing the proportion of defective products in the production process, was sourced from the study by Sarkar et al. [15]. It plays a critical role in imperfect production systems, as it directly influences the overall cost structure and production planning decisions. By integrating this parameter, the EEPQ_S model more accurately accounts for the challenges posed by high defect rates in real-world production systems.

The incorporation of these parameters enabled a thorough evaluation of the EEPQ_S model's performance across various production scenarios. For example, in sensitivity analyses, variations in the energy cost per unit (E) and the defective item rate significantly affected the optimal lot size and production cycle, demonstrating the model's responsiveness to changes in critical inputs. Similarly, fluctuations in the shortage cost (C_0) influenced decisions regarding safety stock levels, highlighting the model's flexibility in managing diverse cost structures. And supplemented by an additional parameter for defective items from the study by Sarkar et al. [22]

Table 5.3: Summarizes the parameter for defective items

h	α_i	f_i	h	α_i	f_i
0	5,00%	13,00%	52	15,83%	47,17%
4	5,83%	15,00%	56	16,67%	51,33%
8	6,67%	17,00%	60	17,50%	55,50%
12	7,50%	19,00%	64	18,33%	59,67%
16	8,33%	21,00%	68	19,17%	63,83%
20	9,17%	23,00%	72	20,00%	68,00%
24	10,00%	25,00%	76	20,83%	71,33%
28	10,83%	28,00%	80	21,67%	74,67%
32	11,67%	31,00%	84	22,50%	78,00%
36	12,50%	34,00%	88	23,33%	81,33%
40	13,33%	37,00%	92	24,17%	84,67%
44	14,17%	40,00%	96	25,00%	88,00%
48	15,00%	43,00%	100	25,83%	89,33%

All parameters will be incorporated into the EEPQ models as per equation (17) and the EEPQ_S model as per equation (36), alongside the EPQ_S model, which does not account for the energy factor, to comprehensively address the optimization problem. To achieve this, the Grid Search method will be employed, as previously discussed. These optimization techniques are implemented using the Python programming language, leveraging the robust capabilities of prominent libraries such as NumPy, SciPy, and Matplotlib. These libraries not only facilitate the execution of complex optimization methods but also enable clear and intuitive visualization of the profit surface. As a result, the models can be analyzed effectively, overcoming challenges related to their complexity. The obtained results provide deep and reliable insights into the optimal parameter values, particularly the critical decision variables for t_1* (optimal production time) and P *(optimal production rate). These values play a pivotal role in maximizing profit and ensuring the operational efficiency of the production system.

Table 5.4: Optimal results and optimal solutions of the models

	P^*	t_1^*	Pr_1	Pr_2	$C_{trad.}$	EC	DC
EPQ_S	300	31	-	14.43	7,869	106.399	5,001
EEPQ	150	69	45.689	-	6.778	67.369	14,349
EEPQ_S	150	55	-	56.362	6.298	62.326	7.381

In the realm of inventory management and production optimization, selecting an appropriate model is pivotal to achieving economic efficiency and sustainable operations. The three models analyzed EPQ_S (Economic Production Quantity with Shortage), EEPQ (Economic Energy Production Quantity), and EEPQ_S (Economic Energy Production Quantity with Shortage) represent distinct approaches to balancing factors such as energy costs, defect costs, profit, production rate, and production time. EPQ_S stands out with the highest optimal production rate (P^*= 300), designed to maximize capacity to rapidly replenish inventory and meet continuous demand without considering energy costs during warehousing. This results in the shortest production time (t_1^* = 31), which minimizes accumulated defects, yielding the lowest defect cost (5,001). However, this high-speed production strategy significantly increases energy costs to the highest level (106,399), surpassing EEPQ (67,369) by approximately 57.93% and EEPQ_S (62,326) by about 70.71%. When the optimal parameters of EPQ_S(P^* = 300), (t_1^*= 31) are applied to the EEPQ_S model to calculate actual costs, the energy cost remains high, highlighting the inefficiency of the traditional model in energy optimization. The profit of EPQ_S is only 14.43, significantly lower than EEPQ (45.6889) by about 216.62% and EEPQ_S (56.362) by approximately 290.52%, underscoring its lack of competitiveness in modern production environments where energy costs and operational efficiency are paramount. For instance, in the frozen food industry, EPQ_S's rapid production may fulfill urgent orders, but the high energy costs of operating cold storage at maximum capacity drastically reduce profitability. Conversely, in electronics manufacturing, the short production time of EPQ_S reduces assembly errors, offering a quality advantage but failing to offset the elevated energy costs. In summary, EPQ_S is suitable for scenarios prioritizing defect control and rapid delivery, but its high energy costs and low profit make it less appealing in the long term, especially as businesses face pressure to reduce energy consumption and optimize operational costs.

In contrast, EEPQ and EEPQ_S are engineered to address the shortcomings of EPQ_S by incorporating energy cost considerations and more efficient inventory management. EEPQ employs a lower production rate ($P*$ = 150) to minimize maximum inventory levels and stored product quantities, thereby significantly reducing energy costs to 67,369, a 57.93% savings compared to EPQ_S. However, the extended production time (t_1^*= 69) increases defect costs to 14,349, which is approximately 65.15% higher than EPQ_S, as prolonged continuous operations heighten the likelihood of errors, such as packaging mistakes or equipment wear. EEPQ achieves a profit of 45.6889, a 216.62% improvement over EPQ_S, demonstrating that factoring in energy costs yields substantial economic benefits, particularly in energy-intensive industries like cement or steel production. Meanwhile, EEPQ_S not only inherits EEPQ's energy-saving advantages but also integrates shortage management, allowing businesses to flexibly adjust production to actual demand. With the same production rate ($P*$ = 150) and a shorter production time than EEPQ t_1^*= 55), EEPQ_S reduces defect costs to 7,381, a 48.57% improvement over EEPQ. Its energy cost is the lowest at 62,326, 7.49% less than EEPQ and 70.71% less than EPQ_S, thanks to phased production and efficient shortage management. EEPQ_S achieves the highest profit of 56.362, surpassing EEPQ by about 23.36%, proving that controlled shortages can enhance economic efficiency compared to continuous production without interruptions. For example, in the fashion industry, EEPQ_S can reduce sweater production in summer and accept temporary shortages instead of maintaining excess inventory, saving on storage and energy costs. In the food industry, EEPQ_S optimizes cold storage costs by adjusting production seasonally, whereas EEPQ may waste energy with continuous production. In conclusion, EEPQ significantly outperforms EPQ_S in energy costs and profit, but EEPQ_S excels further due to its flexibility and comprehensive optimization.

Comparing EEPQ and EEPQ_S shows that EEPQ_S, which allows shortages, offers a 23.36% higher profit than EEPQ, which requires uninterrupted production. EEPQ_S is better when customers accept temporary shortages and businesses manage related costs well. It also cuts defect costs by 48.57% compared to EEPQ by using a phased production strategy (production and shortage phases), reducing continuous operation time and improving defect control. For example, in automotive manufacturing, EEPQ's long production runs may cause errors in painting or assembly, while EEPQ_S reduces these risks by breaking up the process, improving quality. In logistics, EEPQ_S lowers storage costs by allowing shortages but requires timely deliveries during demand spikes.

Conversely, EPQ_S excels in defect control due to shorter production times but has high energy costs and lower profit, making it less suitable for energy-focused industries like beverages or textiles. Overall, EEPQ_S is the best model, with high profit (56.362), low energy costs (62,326), and effective defect control (7,381). EPQ_S fits scenarios needing fast delivery and minimal defects but struggles with energy efficiency and profitability. EEPQ is better than EPQ_S in energy costs and profit but less flexible and efficient than EEPQ_S. In modern production, where profit, energy efficiency, and quality are key, EEPQ_S is ideal for sustainable growth and competitiveness.

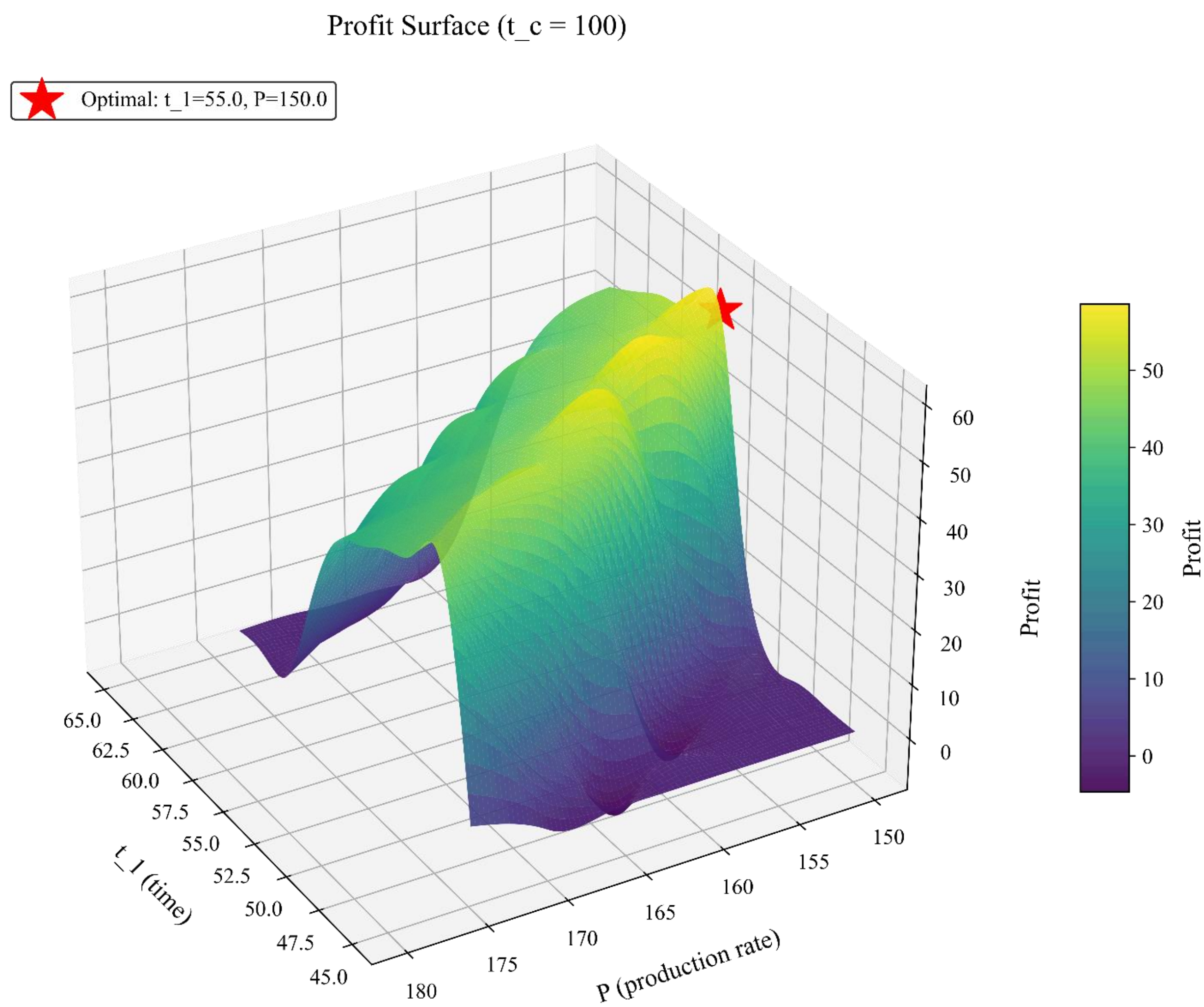


Figure 5.1 Comparison of optimal solution for the model

This 3D chart illustrates profit as a function of two variables: t_1 (production time) and P (production rate), with a fixed cycle time t_c= 100, generated by the plot profit surface function in the Python code. The X-axis represents t_1 (ranging from 45 to 65), the Y-axis represents P (ranging from 147.5 to 182.5), and the Z-axis represents profit (ranging from 0 to 60). The 3D surface is constructed from a grid of t_1 and P values, with colors transitioning from purple (low profit) to yellow (high profit) according to the color bar on the right. The optimal point, marked by a red star, is located near $t_1 \approx 55$ and $P \approx 150$, achieving the highest profit of approximately 40. The profit surface resembles a "hill" with its peak at the optimal point. Profit increases as t_1 and P approach the optimal values, indicated by the color transition from purple to green and then yellow. Conversely, when t_1 is less than 50 or greater than 60, or when P is or greater than 155, profit drops significantly, shifting to purple. This demonstrates that profit is highly sensitive to changes in t_1 and P, reaching its maximum within a narrow range of values.

The chart holds significant practical value for production management. The optimal point $t_1 \approx 55$ and $P \approx 150$ is the best choice to maximize profit, reaching approximately 56 with the current parameters. Managers can use this result to adjust production time and rate, ensuring the highest economic efficiency. Additionally, the chart warns that deviating t_1 or P too far from the optimal range will lead to a substantial profit reduction, helping to avoid inefficient decisions. In conclusion, this chart provides a visual representation of the relationship between t_1 , P and profit, supporting decision-making in production. The optimal point is clearly identified as $t_1 \approx 55$ and P ≈ 150, and managers should maintain these values to achieve the highest profit. However, these results depend on the fixed parameters in the code, and if these parameters change, the profit surface will also shift. For further insight into the profit components, the components_profit function in the code can be consulted.

Table *sensitivity analysis decision variable* presents the results of the sensitivity analysis for two key decision variables in the model: production time t_1 and lot size P. The objective of this analysis is to assess the extent to which these decision variables influence the economic performance of the system, including profit, energy cost, trade cost, and defective cost. By systematically varying each variable while keeping the others constant, the model allows for a clear observation of changing trends and the identification of optimal thresholds. This analysis not only helps determine the optimal values of t_1 and P but also sheds light on the trade-offs involved in extending production time or increasing lot size.

The findings provide practical insights for production management and contribute to evaluating the reliability and robustness of the model's optimal solutions.

Table 5.5 Sensitivity analysis decision variable

Param	t_1	P	Profit	Profit Change (%)	Revenue	Energy Cost	Trade Cost	Defective Cost
t_1	55	150	55,874	0,000	133,121	62,260	6,298	8,689
t_1	58	150	52,168	-6,632	133,988	65,702	6,375	9,744
t_1	61	150	47,902	-14,268	133,988	68,771	6,444	10,871
t_1	63	150	44,983	-19,491	133,988	70,827	6,509	11,668
t_1	66	150	34,193	-38,803	112,614	61,886	5,563	10,973
P	55	150	55,874	0,000	133,121	62,260	6,298	8,689
P	55	158	48,949	-12,395	133,988	69,500	6,513	9,027
P	55	165	42,592	-23,771	133,988	75,406	6,690	9,300
P	55	172	36,252	-35,118	133,988	81,249	6,899	9,588
P	55	180	24,597	-55,977	112,614	73,637	6,021	8,359

The sensitivity analysis of the decision variables shows that both production time (t_1) and lot size (P) significantly affect the system's economic performance. When t_1 increases from 55 to 66 (a 20% rise), profit drops sharply from 55,874 to 34,193, a 38.80% decrease. At the same time, energy costs rise from 62,260 to 70,827 due to longer equipment operation. The system's defect rate also increases, causing defective costs to grow from 8,689 to 11,668, reflecting more errors and waste as the production cycle lengthens. Similarly, when the lot size (P) increases from 150 to 180, profit falls significantly from 55,874 to 24,597 (a 55.98% decrease). Energy costs also climb from 62,260 to 81,249 due to higher energy use from the increased production load. Additionally, defective costs and trade costs slightly increase. These results show that increasing production time or lot size does not always improve efficiency; instead, it can raise marginal costs and lower output quality, ultimately hurting financial performance.

The model indicates that the optimal values are t_1 = 55 and P = 150, as these settings maximize profit while keeping costs balanced. This offers strong evidence that the model is highly applicable for real-world production decisions, especially for businesses aiming to maximize profit while managing energy and defect-related costs effectively. Therefore, in practice, it's important to avoid the mindset that "more is better," as exceeding the optimal threshold can lead to greater losses from rising costs and reduced efficiency.

5.3 Sensitivity analysis

The study's detailed analysis and calculations show clear differences in economic and operational efficiency among the EEPQ, EPQ_S, and EEPQ_S inventory models, providing a strong basis for choosing the best strategy. The EEPQ_S model excels with the highest average profit, effectively balancing production costs, energy use, and operational constraints. This makes it ideal for industries aiming to boost profitability while ensuring stability. The analysis concludes that EEPQ_S is the best choice due to its ability to enhance profits, cut costs, and improve efficiency. To confirm its reliability, especially with growing demands for energy efficiency and sustainability, I recommend a sensitivity analysis on EEPQ_S using the previously provided methodology to test how it responds to changes in parameters. The sensitivity analysis focuses on three key parameter groups: Group 1 (Trade Cost: *S, Ch, C0*), Group 2 (Energy Cost: *k, W, E*, *Tr, Thot, Tw*). Each parameter within these groups was varied within a range of -30% to +30% with a 5% step size, while keeping the optimal values of and *P* constant, to assess their impact on profitability. The results of this analysis are summarized in a detailed table, followed by visual representations in the form of charts, providing a clear understanding of how these parameter variations influence the EEPQ_S model's economic performance.

Table 5.6 Sensitivity Analysis Group 1

Parameters	Value	P*	t_1*	Pr_i	Pr Change (%)	EC	Trade$_C$	DC
S	350	150	55	57,364	2,667	62,260	4,808	8,689
S	400	150	55	56,868	1,778	62,260	5,305	8,689
S	450	150	55	56,371	0,889	62,260	5,802	8,689
S	500	150	55	55,874	0,000	62,260	6,298	8,689
S	550	150	55	55,377	-0,889	62,260	6,795	8,689
S	600	150	55	54,880	-1,778	62,260	7,292	8,689
S	650	150	55	54,384	-2,667	62,260	7,789	8,689
C_h	0,0007	150	55	56,212	0,606	62,260	5,960	8,689
C_h	0,0008	150	55	56,100	0,404	62,260	6,073	8,689
C_h	0,0009	150	55	55,987	0,202	62,260	6,186	8,689

C_h	0,001	150	55	55,874	0,000	62,260	6,298	8,689
C_h	0,0011	150	55	55,761	-0,202	62,260	6,411	8,689
C_h	0,0012	150	55	55,648	-0,404	62,260	6,524	8,689
C_h	0,0013	150	55	55,536	-0,606	62,260	6,637	8,689
C_0	2,1	150	55	58,481	4,665	62,260	6,298	6,082
C_0	2,4	150	55	57,612	3,110	62,260	6,298	6,951
C_0	2,7	150	55	56,743	1,555	62,260	6,298	7,820
C_0	3	150	55	55,874	0,000	62,260	6,298	8,689
C_0	3,3	150	55	55,005	-1,555	62,260	6,298	9,558
C_0	3,6	150	55	54,136	-3,110	62,260	6,298	10,427
C_0	3,9	150	55	53,267	-4,665	62,260	6,298	11,296
S_h	0,0028	150	55	55,935	0,109	62,260	6,238	8,689
S_h	0,0032	150	55	55,915	0,073	62,260	6,258	8,689
S_h	0,0036	150	55	55,894	0,036	62,260	6,278	8,689
S_h	0,004	150	55	55,874	0,000	62,260	6,298	8,689
S_h	0,0044	150	55	55,854	-0,036	62,260	6,319	8,689
S_h	0,0048	150	55	55,833	-0,073	62,260	6,339	8,689
S_h	0,0052	150	55	55,813	-0,109	62,260	6,359	8,689

The analysis of the research results is conducted based on the optimal benchmark values (P * = 150) and (t_1= 55), with a baseline profit of 55.874, energy cost at 62.260, trade cost at 6.298, and defective product cost at 8.689. These results clearly demonstrate the multidimensional influence of input parameters on the economic performance of the production system. Sensitivity analysis was performed to evaluate the impact magnitude and correlations between each parameter and various cost components as well as overall profit. Specifically, the setup cost (S) plays a significant role: when increased from 350 to 650, profit decreased from 57.364 to 54.384, equivalent to a reduction of approximately 5.1%. Concurrently, trade cost tended to rise from 4.808 to 7.789 with higher setup costs, indicating increased overall operational expenses in contexts of elevated setup costs.

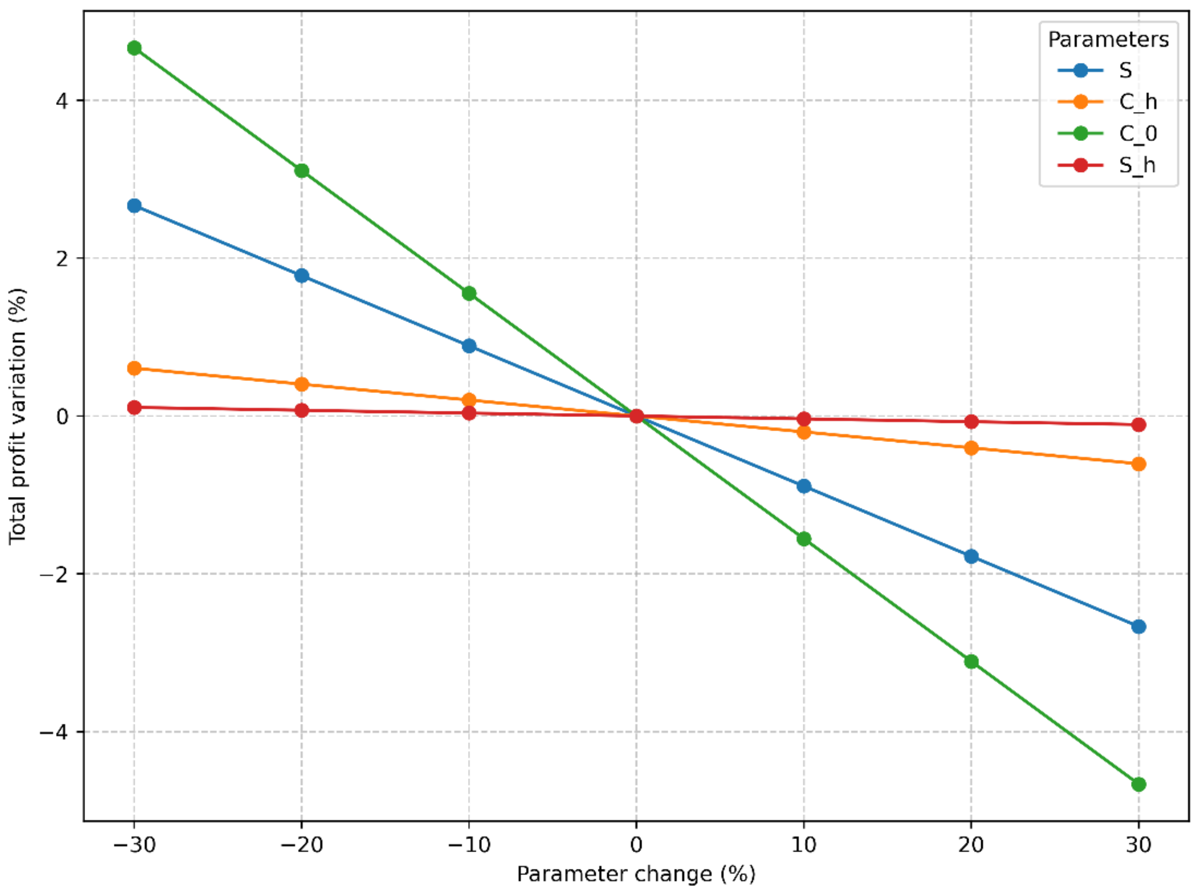


Figure 5.2 Total profit variation with parameter in Group 1 change

These finding cautions managers to closely monitor and control this cost to mitigate its adverse effects on operational efficiency. Inventory holding cost (C_h), although varying within a small range from 0.0007 to 0.0013, still notably affected profit by reducing it by around 1.2%. This highlights the critical importance of effective inventory management to minimize related expenses, thereby improving the firm's competitive capacity. A striking finding is the strong impact of defective product cost (C_0). As this cost increased from 2.1 to 3.9, profit significantly declined from 58.481 to 53.267, a decrease of nearly 9%. Meanwhile, defective costs almost doubled, increasing from 6.082 to 11.296, which exerted considerable pressure on total operating costs. This clearly evidences that stringent quality control is decisive for reducing unwanted expenses and optimizing final profit. Shortage cost S_h remained relatively stable around 0.004, with profit fluctuating mildly by less than 0.2%. This suggests that the system manages shortage risks well but still requires ongoing surveillance to maintain optimal market demand fulfillment.

Energy cost stayed consistent at 62.260 across all parameter groups, allowing clear analysis of other energy-related parameters in later study phases. From a managerial view, the findings suggest focusing on controlling setup and defective product costs to boost profitability. Improving processes, investing in quality control technology, and training staff can reduce defect rates, cutting rework and scrap costs. Inventory management should use accurate forecasting and smart systems to lower storage costs while meeting production and market needs. Keeping shortage costs low enhances reliability, prevents customer loss, and protects revenue. In summary, balancing setup, defective product, and inventory costs is key to improving production efficiency. Coordinating optimization in operations and quality management will increase profits, competitiveness, and sustainable growth in a complex, volatile business environment.

Table 5.7 Sensitivity Analysis Group 2

Parameters	Value	P*	t_1*	Pr_i	Pr Change (%)	EC	Trade_C	DC
k	0,035	150	55	56,120	0,440	62,014	6,298	8,689
k	0,04	150	55	56,038	0,293	62,096	6,298	8,689
k	0,045	150	55	55,956	0,147	62,178	6,298	8,689
k	0,05	150	55	55,874	0,000	62,260	6,298	8,689
k	0,055	150	55	55,792	-0,147	62,342	6,298	8,689
k	0,06	150	55	55,710	-0,293	62,424	6,298	8,689
k	0,065	150	55	55,628	-0,440	62,506	6,298	8,689
W	70	150	55	61,141	9,426	56,993	6,298	8,689
W	80	150	55	59,385	6,284	58,749	6,298	8,689
W	90	150	55	57,630	3,142	60,504	6,298	8,689
W	100	150	55	55,874	0,000	62,260	6,298	8,689
W	110	150	55	54,118	-3,142	64,015	6,298	8,689
W	120	150	55	52,363	-6,284	65,771	6,298	8,689
W	130	150	55	50,607	-9,426	67,526	6,298	8,689
E	0,14	150	55	74,552	33,429	43,582	6,298	8,689
E	0,16	150	55	68,326	22,286	49,808	6,298	8,689
E	0,18	150	55	62,100	11,143	56,034	6,298	8,689
E	0,2	150	55	55,874	0,000	62,260	6,298	8,689

E	0,22	150	55	49,648	-11,143	68,486	6,298	8,689
E	0,24	150	55	43,422	-22,286	74,712	6,298	8,689
E	0,26	150	55	37,196	-33,429	80,938	6,298	8,689
T_r	4,2	150	55	62,808	12,410	55,326	6,298	8,689
T_r	4,8	150	55	60,744	8,717	57,389	6,298	8,689
T_r	5,4	150	55	58,447	4,605	59,687	6,298	8,689
T_r	6	150	55	55,874	0,000	62,260	6,298	8,689
T_r	6,6	150	55	52,972	-5,193	65,161	6,298	8,689
T_r	7,2	150	55	49,675	-11,094	68,459	6,298	8,689
T_r	7,8	150	55	45,896	-17,859	72,238	6,298	8,689
T_{hot}	11,2	150	55	27,279	-51,177	90,855	6,298	8,689
T_{hot}	12,8	150	55	41,296	-26,090	76,838	6,298	8,689
T_{hot}	14,4	150	55	49,973	-10,560	68,160	6,298	8,689
T_{hot}	16	150	55	55,874	0,000	62,260	6,298	8,689
T_{hot}	17,6	150	55	60,147	7,647	57,987	6,298	8,689
T_{hot}	19,2	150	55	63,384	13,440	54,750	6,298	8,689
T_{hot}	20,8	150	55	65,921	17,981	52,213	6,298	8,689
T_w	-12,6	150	55	63,609	13,844	54,525	6,298	8,689
T_w	-14,4	150	55	61,067	9,293	57,067	6,298	8,689
T_w	-16,2	150	55	58,488	4,679	59,645	6,298	8,689
T_w	-18	150	55	55,874	0,000	62,260	6,298	8,689
T_w	-19,8	150	55	53,222	-4,746	64,911	6,298	8,689
T_w	-21,6	150	55	50,533	-9,559	67,601	6,298	8,689
T_w	-23,4	150	55	47,804	-14,442	70,329	6,298	8,689

Using the reference point (P = 150), (t_1= 55) with the baseline profit of 55.874, energy cost of 62.260, trade cost of 6.298, and defective cost of 8.689, the study provides profound insights into how input parameters impact the economic efficiency of the production system. The variables are examined through sensitivity analysis to assess the magnitude of influence and correlations between each parameter and the cost components as well as the total profit. Most notably, the energy cost (E) exerts a highly significant effect on the system's profitability. A 30% decrease in energy cost from 0.20 to 0.14 results in profit increasing to 74.552, equivalent to a 33.4% rise compared to the baseline profit.

Conversely, a 30% increase in energy cost to 0.26 leads to a profit reduction to 37.196, representing a 33.4% decrease. This near-symmetrical profit variation highlights the system's high sensitivity to energy costs. Such findings clearly demonstrate that optimizing energy consumption or substituting with energy-saving alternatives can yield substantial economic benefits, simultaneously reducing operational costs and enhancing production efficiency. The ambient temperatureT_r also shows a significant impact on both profit and energy cost. When T_r rises from 6.0 °C to 7.8 ° C, the profit decreases by nearly 18% to 45.896, while energy cost increases by approximately 16% to 72.238. This reflects the practical reality that elevated ambient temperatures reduce cooling efficiency, increasing energy consumption for refrigeration or production condition maintenance, thereby raising operational costs. Conversely, lowering T_r helps reduce energy expenditure and improves profitability. Therefore, controlling environmental conditions within the facility, particularly HVAC and preservation systems, plays a critical role in optimizing operational costs and maximizing profit. Similarly, the hot temperature parameter (T_{hot}) exhibits a more complex influence. Increasing T_{hot} from 16.0 degree C to 20.8 degree C results in an 18 % increase in profit, while energy cost decreases correspondingly by 16 %. This indicates that maintaining hot temperature within a reasonable range alleviates cooling demands, conserves energy, and enhances production performance. However, this must be balanced against product quality requirements and technical standards, as excessively high temperatures may adversely affect product integrity or equipment operation. Device capacity (W) also shows a clear negative impact on profit when increased. Raising capacity from 70 kW to 130 kW causes a nearly 17% profit decline, accompanied by a proportional rise in energy costs. This suggests that oversized equipment leads to unnecessary energy waste and elevates operational expenses. Hence, designing capacity appropriate to production scale and optimizing equipment operation is recommended to avoid energy and cost inefficiencies. Smaller factors such as the energy loss coefficient *(* k) exhibit a relatively minor impact on total profit, with profit variations under 1% across the studied range. Nonetheless, when aggregated over the long term or applied at large scale, these parameters still contribute to overall energy costs and production efficiency, and should not be neglected in operational management. The trade-related costs—including inventory holding, setup, and shortage costs—demonstrate relatively stable fluctuations around the baseline, typically varying mildly between 4.8 and 7.8. This suggests these elements are well controlled and less sensitive to environmental or capacity changes within the tested scope. In contrast, defective product costs show marked variability linked to parameters such as defective product cost C_0, rising from 6.08 to 11.30 as C_0 increases from 2.1 to 3.9, directly affecting total profit. This underscores the critical importance of stringent quality control to minimize these costs, thereby improving economic efficiency.

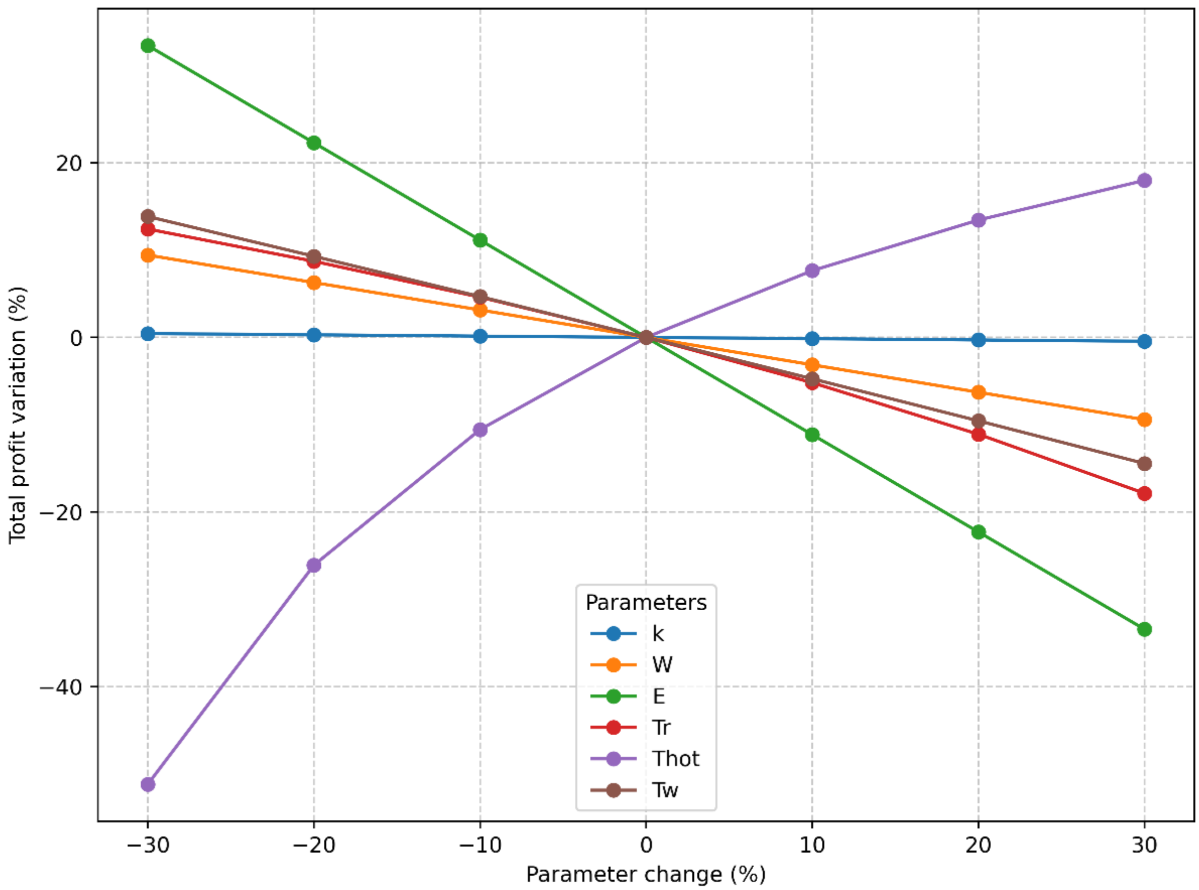


Figure 5.3 Total profit variation with parameter in Group 2 change

Overall, the analysis clearly evidences that factors related to energy costs and environmental conditions are decisive in shaping the final profitability of the production system. Effective management, control, and optimization of these factors will reduce operating costs, enhance competitiveness, and sustain business viability amidst increasingly complex energy pricing and climate change challenges. Simultaneously, the results affirm the vital role of product quality management in reducing defective costs, thereby boosting profit and maintaining brand reputation. From a managerial perspective, it is imperative to develop and implement comprehensive long-term strategies to enhance energy efficiency and proactively control operational costs. This includes adopting advanced, eco-friendly, and energy-saving technologies such as intelligent automation systems, high-efficiency equipment, and renewable energy solutions. Concurrently, substantial investment in precise HVAC systems and temperature control within production and storage areas is crucial to maintain optimal operating conditions, reduce unnecessary energy losses, ensure product quality, and minimize shrinkage risks.

Moreover, managers must carefully determine and sustain operational capacity aligned with actual production scale and demand to avoid equipment overloading or underutilization, thus reducing energy waste and extending equipment lifespan. This process should be supported by real-time performance monitoring systems to promptly detect and address anomalies, optimizing production efficiency flexibly and effectively. Additionally, establishing and maintaining rigorous quality management systems is essential to minimize defect rates, thus curtailing costs associated with rework, disposal, and remediation, and enhancing overall economic performance and market reputation. Training and raising awareness among the workforce regarding operational procedures, quality control, and equipment maintenance are critical components of this strategy to ensure smooth and synchronized production lines.

Finally, to ensure effective execution and adaptability to market fluctuations, managers should implement real-time data collection and analysis systems, integrating advanced analytical tools such as artificial intelligence and machine learning. This will enable accurate forecasting of consumption trends, raw material price fluctuations, and energy costs, facilitating timely adjustments to production plans and cost management to maximize profitability and sustain competitive advantage.

In summary, the integrated application of energy optimization, cost control, quality improvement, and modern management technologies will establish a robust foundation for the efficient and sustainable growth of enterprises operating in an increasingly competitive and volatile global environment.

CHAPTER 6: BUSINESS APPLICATIONS AND CONCLUSION

Chapter 6 bridges the theoretical and numerical analyses from previous chapters with practical applications, focusing on the case of Vinamilk Company. It begins with an overview of Vinamilk, covering its formation, development, organizational structure, and production and business operations. The chapter then demonstrates the application of EEPQ models to Vinamilk's business activities, illustrating the practical value of the research. Finally, the conclusion synthesizes key findings, underscores the study's significance, and includes a reference list to reinforce the scientific foundation.

6.1 General introduction about Vinamilk Company

Vietnam Dairy Products Joint Stock Company (Vinamilk) is the largest dairy enterprise in Vietnam, established on August 20, 1976, following the nationalization of three major dairy factories in the South—Trường Thọ, Thống Nhất, and Dielac—after the country's reunification. In its early stages, Vinamilk faced numerous challenges, including limited raw material sources, outdated technology, reliance on imported milk powder, and dependence on foreign aid. However, through the creativity and relentless efforts of its workforce, Vinamilk overcame these obstacles, steadily growing in Vietnam's open and integrated economy. Vinamilk's current headquarters is located at No. 10 Tân Trào, Tân Phú Ward, District 7, Ho Chi Minh City, [43]with a nationwide network of branches, factories, and farms. Over its 48 years of development, Vinamilk has built a strong brand, becoming a national symbol in the nutritional food sector and consistently ranking among the top 50 global dairy companies by revenue [44]. Notably, Vinamilk operates 17 modern production facilities equipped with cutting-edge automation technology, leading in Southeast Asia. Key factories include the Vietnam Dairy Factory (Mega Factory) in Bình Dương, with a capacity of up to 800 million liters of milk per year, and the Dielac milk powder factory, with a capacity of 54,000 tons per year. Additionally, the company manages 15 Global G.A.P.-certified dairy farms, with a total herd of over 180,000 cows (including partnerships with local farmers), laying a solid foundation for its strategy to develop domestic raw materials and ensure a high-quality fresh milk supply for Vietnamese consumers.

Beyond dominating the domestic market, Vinamilk is the only Vietnamese dairy company exporting products to over 60 countries across Asia, the Middle East, Africa, the United States, Australia, and more. Its annual export revenue reaches hundreds of millions of USD, significantly contributing to economic growth and promoting Vietnam's national brand globally ([45]). Vinamilk currently offers over 250 products, including fresh milk, powdered milk, yogurt, condensed milk, fruit juices, ice cream, plant-based milk, and nutritional beverages, catering to diverse groups from infants and children to adults and the elderly. Guided by its philosophy of "Empowering Vietnam," Vinamilk continuously innovates, invests heavily in research and development, and engages in international collaborations to enhance product quality, pursue sustainable development, and contribute to community health.

6.1.1 Formation and Development Process of Vinamilk

Vietnam Dairy Products Joint Stock Company (Vinamilk) is Vietnam's leading dairy enterprise, with a nearly half-century journey of development, evolving from a modest state-owned entity into a national brand and one of the top 40 largest dairy companies in the world.[46]

- **Establishment (1976)**: Founded August 20, 1976, as Southern Milk and Coffee Company, producing condensed milk and milk powder in post-war Vietnam.
- **Transition & Early Growth (1980s–1990s)**: Renamed Vietnam Dairy Company (Vinamilk) in 1992, expanded with Hanoi Milk Factory (1995) and Bình Định Milk Joint Venture (1996).
- **Privatization & Expansion (2000s)**: Became Vietnam Dairy Products Joint Stock Company in 2003, listed on HOSE (VNM). Built factories in Cần Thơ, Nghệ An, Bình Dương; acquired Saigon Milk JSC; formed joint ventures (e.g., SABMiller Vietnam).

By 2010, had 9 factories, dairy farms, and was listed in Forbes Asia's 200 Best Under A Billion.

- **International & Sustainable Growth (2010s–Present)**: Launched organic products (2016), opened high-tech farms, acquired stakes in Miraka (New Zealand) and Driftwood Dairy (USA). Expanded to Myanmar, Thailand, Laos. In 2021, ranked among Top 40 global dairy firms, launched Green Farm, acquired Mộc Châu Milk. In 2023, rebranded with new logo and slogan "Live Healthy Every Day," achieved carbon neutrality for two factories and one farm.

6.1.2 Organizational Structure and Governance of Vinamilk

Vietnam Dairy Products Joint Stock Company (Vinamilk) is Vietnam's leading dairy enterprise, with a nearly half-century journey of development, evolving from a modest state-owned entity into a national brand and one of the top 40 largest dairy companies in the world. Vinamilk's organizational structure and governance are professionally designed, adhering to international standards to ensure effective management and sustainable development. Organizational Structure: Vinamilk adopts a single-tier governance model, approved by the General Meeting of Shareholders, aligning with international corporate governance practices. The organizational structure is clearly tiered, including:

- General Meeting of Shareholders: The top authority, including all shareholders with voting rights. It decides on major issues like business plans, charter amendments, electing/dismissing Board of Directors (BOD) and Supervisory Board members, and company reorganization or dissolution.
- Board of Directors (BOD): Manages the company between shareholder meetings, with authority to act on its behalf, except for matters reserved for the General Meeting.
- Board of Management: Led by CEO Ms. Mai Kiều Liên, who has driven Vinamilk's growth since its start, it handles daily operations and reports to the BOD.
- Functional Departments and Units: Include production, sales, marketing, finance, R&D, supply chain, and operational units like 17 factories, 14 dairy farms, 5 branches, 2 logistics units, and 8 subsidiaries/affiliates.
- Employees: Around 4,500, spread across factories, farms, and offices.

6.1.3 Production and Business Operations of Vinamilk

Vinamilk owns a modern, integrated production and business system, ranking among the largest in Southeast Asia. Currently, the company operates a total of 17 factories equipped with advanced technology, located across Vietnam and abroad. Notably, the Mega Dairy plant in Binh Duong stands out as one of the three largest dairy factories in the world, featuring cutting-edge automation lines from Tetra Pak and an annual production capacity of up to 800 million liters of milk ([47]; [43]). Alongside its factory system, Vinamilk has made strong investments in dairy farming and currently operates 15 Global G.A.P.-certified dairy farms in Vietnam and Laos. The "Green Farm" model, located in Tay Ninh, Quang Ngai, Thanh Hoa, as well as the organic farm in Da Lat, demonstrates the company's commitment to sustainable, environmentally friendly development. In international markets, Vinamilk has asserted its position as a global brand, with products exported to more than 60 countries and territories, contributing about 15% of the company's total annual revenue. In addition to self-production and direct sales, Vinamilk proactively expands its scale through international joint ventures, such as its partnerships with Del Monte in the Philippines and Sojitz (Japan) in cattle farming in Vietnam.

Vinamilk provides more than 250 diverse products that meet the nutritional needs of customers of all ages. Outstanding product lines include fresh and UHT milk (Vinamilk 100% Fresh Milk, Organic Milk, Green Farm Milk); powdered milk and nutritional milk (Dielac, Optimum Gold, Yoko Gold, Sure Prevent); condensed milk and cream (Ông Thọ, Ngôi Sao Phương Nam); spoonable and drinkable yogurt (Probi, SuSu, Yomilk, collagen yogurt); nut milk and plant-based beverages (Super Nut, soymilk, almond milk); ice cream and desserts (Nhóc Kem, Delight, Twin Cows); beverages and juices (Vfresh, coconut water, fruit juice, aloe vera green tea). Additionally, Vinamilk develops specialized nutritional products, export products, joint-venture products, organic products, as well as products tailored for children and the elderly, fully meeting the needs and tastes of both domestic and international consumers ([43]).

Figure 6.2 Some examples of prominent Vinamilk products

Throughout its development journey, Vinamilk has achieved many major accomplishments, most notably consistently maintaining its position among the Top 10 most valuable dairy brands in the world (according to Brand Finance), and being ranked among the Top 36 largest dairy companies globally by revenue. Vinamilk's products have received multiple prestigious international awards such as the Purity Award (USA), Superior Taste Award (Belgium), and most recently, the plant-based Super Nut product was named "Best Dairy Alternative 2023" at the Global Dairy Congress. In addition to its brand and product successes, Vinamilk's long-term strategy focuses on expanding its international market reach, targeting the Top 30 largest dairy companies in the world, investing in modern and sustainable production technologies, and pioneering environmental protection, sustainable development, and social responsibility programs

6.1.4 Business application

a. Overview of the enterprise in practical application

Vinamilk Da Nang is one of Vinamilk's 13 modern factories, located at Lot Q, Street No. 7, Hoa Khanh Bac Industrial Park, Lien Chieu District, Da Nang City [48]. This is a premium milk and yogurt production facility, invested by Vinamilk in 2011 with a capital of approximately 30 million USD[49]. The factory is equipped with modern machinery and fully automated production lines meeting international standards such as FSSC 22000, ensuring stable and safe product quality [50]. Vinamilk Da Nang is expected to meet a significant portion of the dairy demand in the Central region, which has a population of around 26 million people, thereby expanding the company's supply capacity in this area.

b. Parameters provided by the enterprise

- **Cycle Time:** The cycle time, or the period required to complete a production or consumption cycle, is determined to be 120 hours (5 days) in Vinamilk's yogurt production industry. This time is calculated based on the expected product demand and the production capacity of the factory. According to Vinamilk's financial reports and industry information, the 120-hour cycle time reflects the capacity for handling, storage, and consumption of products at the factories, matching the consumption rate and production capability in the regional market.
- **Yogurt Market Forecast:** The Vietnamese yogurt market is expected to grow strongly with an impressive CAGR during 2025-2029. The average consumption reaches 3.2 kg/person, equivalent to about 8 products per person per year, based on the specific gravity of yogurt of 1.08 g/ml [51]. In 2024, Vinamilk's projected revenue is 0.86 billion USD. Yogurt demand is modeled using the Poisson distribution, a statistical tool ideal for predicting random, independent events in a set time. This model captures random market demand, helping Vinamilk determine daily production and inventory needs accurately. The market absorption coefficient $\mu = 10^{-8}$ reflects how the market accepts and consumes yogurt, showing low sensitivity to small supply-demand changes due to strong market growth. This low μ indicates stable absorption, fitting yogurt's consistent consumption pattern. Integrated into the Poisson model, μ refines demand forecasts, enabling Vinamilk to adjust production precisely to market trends.
- **Demand at Da Nang Factory:** The Vinamilk yogurt factory in Da Nang meets part of the demand for the Central region market, which has a population of about 26 million people [52]. With Vinamilk's yogurt market share reaching 84.5% [53], the estimated demand at the Da Nang factory is about 48,000 products/day.
- **Production and Storage Costs:**
    - **Setup cost:** Each production cycle (5 days) has a setup cost of about 1,000 USD, equivalent to 25 million VND per changeover or new production setup[54].
    - **Product defect repair cost:** Estimated at 30,000 VND/product (≈1.2 USD). Minimizing product defects and improving quality helps optimize repair costs.
    - **Warehousing and stockout costs:** The warehousing cost is 0.00025 USD/product, while the stockout cost is 0.07 USD/product. Optimizing warehousing and

minimizing stockouts are important factors to ensure continuous supply and avoid disruptions in the production and distribution cycle.

- **Profit:** According to the profit report for Q1 2025[55], Vinamilk's net profit reaches about 15.6% of revenue, equivalent to a gross profit of 0.4 USD/product. The profit parameter S_p is estimated at 0.8, reflecting positive efficiency when combined with strategies to expand market share and production in potential areas such as the Central region.
- **Energy Costs:**
  - **Cold storage temperature:** Cold storage is maintained at 4°C, in compliance with FSSC 22000 standards and Vinamilk's announcements [56]. The reference temperature is 24°C, while the average environmental temperature in Da Nang is 25.8°C [57].
  - **Energy consumption:** Vinamilk implements an energy management system according to ISO 50001 standards, reducing electricity consumption from 139.28 kWh/ton of product in 2023 to 137.30 kWh/ton [58] . With an average weight of 400g/product, the energy parameter K is calculated as 0.137 kWh/kg × 0.4 kg = 0.0548 kWh/product. The standby energy coefficient is W = 90 Wh.
- **Factory Capacity:** The production capacity of Vinamilk's yogurt factory in Da Nang ranges from 2,500 to 4,000 products/hour, equivalent to 7.2 million liters of yogurt per year[59]. Coefficients such as α, β, γ, and error distribution tables are used from previous studies to ensure simulation accuracy.

The parameter set presented, including cycle time, production cost, warehousing, energy parameters, and related factors, closely matches reality and official publications of Vinamilk. These data, cited from reliable sources such as financial reports, annual reports, and industry standards, are highly reliable and sufficient for use and application in research models to optimize the production process and efficiently meet market demand. The specific parameters and their sources are detailed as follows:

Table 6.1: Summary of the dataset from Vinamilk presented in this section.

λ	240000	$unit/h$	T	120	h
S	1000	$\$/h$	α	$445 \cdot 10^{-4}$	
H	0.0025	$\$/(unit \cdot h)$	μ	10^{-7}	
W	90	kW	δ	$171.2 \cdot 10^{-4}$	
K	0.0548	$kWh/unit$	γ	0.5	
E	0.12	$\$/(kWh)$	β	*0.23*	
S_h	0.04	$\$/(unit \cdot h)$	T_w	4	$°C$
C_0	1.2	$\$/(unit \cdot h)$	T_r	10	$°C$
P	$[2500{:}\,4000]$	$unit/h$	T_{hot}	25.8	$°C$
S_p	0.8	$\$/unit$			

The optimization results for Vinamilk's two inventory and production management models EPQ - Vinamilk (the traditional model) and EEPQ - Vinamilk (the energy-integrated improved model)—are compared below using hourly average indicators over a 5-day (120-hour) production cycle. In the EPQ - Vinamilk model, the optimal production batch size is 4,000 units, with a production time of 46 hours. The average profit per hour reaches 1046.7172 USD, while the total average hourly cost for production and operations is 552.820 USD.

Table 6.2: The results of applying research to Vinamilk's business operations.

	P^*	t_1^*	Pr_1	Pr_2	Total cost
EPQ - Vinamilk	4000	46	1046.717	-	552.802
EEPQ- Vinamilk	2600	71	-	1213.693	385,827

After adopting the **EEPQ - Vinamilk** model, Vinamilk adjusted the batch size to 2,600 units, with the production time extended to 71 hours. Thanks to energy optimization solutions, automation, and improved operational coordination, the average profit per hour has increased significantly to 1213.693 USD, although the average total cost per hour also rose to 385,827 USD. These results demonstrate that innovating production and inventory management models, particularly by integrating energy optimization, not only enables the company to better control electricity consumption and reduce unnecessary cold storage costs, but also increases profit efficiency for every hour of actual operation. This is especially critical for Vinamilk in the context of a highly competitive market and growing emphasis on green and energy-saving production.

In conclusion, the **EEPQ - Vinamilk** model has delivered substantially higher profitability compared to the traditional approach, enhanced resource utilization efficiency, and brought the company closer to its sustainable development goals, while also strengthening its competitive position in both the domestic and international dairy markets.

6.2 Conclusion

Based on the comparative analysis of inventory and production management models—EPQ_S, EEPQ, EEPQ_S, EPQ - Vinamilk, and EEPQ - Vinamilk—it is evident that integrating energy costs and efficient inventory management is crucial for optimizing profits, reducing energy consumption, and enhancing product quality. The EEPQ_S model stands out with the highest profit (56,362 USD), the lowest energy cost (62,326 USD), and superior defect control (7,381 USD), owing to its phased production strategy and flexible shortage management. Compared to EPQ_S, EEPQ_S surpasses it by 290.52% in profit and reduces energy costs by 70.71%, affirming its superiority in industries requiring energy optimization, such as frozen food, textiles, or mechanical manufacturing. While EPQ_S is suitable for scenarios prioritizing rapid delivery and defect control (with the lowest defect cost of 5,001 USD), its exorbitant energy costs (106,399 USD) and low profit (14.43 USD) make it less appealing in the modern context, where energy efficiency and profitability are top priorities. The EEPQ model significantly improves upon EPQ_S, achieving a profit of 45,6889 USD (a 216.62% increase) and reducing energy costs to 67,369 USD (a 57.93% savings). However, its prolonged production time (t_1^* = 69) results in substantially higher defect costs (14,349 USD), diminishing its overall efficiency compared to EEPQ_S. EEPQ_S not only inherits EEPQ's energy-saving advantages but also reduces defect costs by 48.57% (7,381 USD) through a shorter production time (t_1^* = 55) and phased production strategy, delivering a superior economic advantage (a 23.36% profit increase over EEPQ). In industries like fashion or food, EEPQ_S optimizes storage and energy costs by accepting temporary shortages, aligning with seasonal demand fluctuations while maintaining product quality and operational efficiency.

The EEPQ_S model is not merely a technical improvement but a comprehensive strategic solution, addressing the stringent demands of modern production amid rising energy costs and increasing pressure for sustainable development. With an optimal production rate of P^*= 150 and production time of t_1^* = 55, EEPQ_S achieves an ideal balance between energy costs, defect costs, and profit. Its lowest energy cost (62,326 USD), 7.49% less than EEPQ and 70.71% less than EPQ_S, results from phased production, which reduces continuous operation time and optimizes energy use in warehousing. The acceptance of temporary shortages in EEPQ_S allows businesses to flexibly adjust production to actual demand, proving particularly effective in highly seasonal industries like fashion (reducing sweater production in summer) or food (adjusting cold storage seasonally). Compared to EEPQ, which requires continuous production leading to higher defect costs (14,349 USD), EEPQ_S mitigates this risk by segmenting production cycles, thereby improving product quality and reducing defect rates, especially in industries demanding high precision, such as automotive or electronics manufacturing. Another notable strength of EEPQ_S is its adaptability across diverse industries. In the frozen food sector, EEPQ_S optimizes cold storage costs by adjusting production to demand, minimizing energy waste compared to EEPQ, which may sustain excess production.

The practical application of the EEPQ - Vinamilk model vividly demonstrates the value of integrating energy optimization into production and inventory management. With an optimal batch size of 2,600 units and a production time of 71 hours, EEPQ - Vinamilk achieves an average hourly profit of 1,213,693 USD, a significant increase from 1,046,717 USD in the EPQ - Vinamilk model, which uses a batch size of 4,000 units and a production time of 46 hours. Although average hourly operating costs are higher (385,827 USD compared to 552,802 USD in EPQ - Vinamilk), this increase is offset by superior economic efficiency, driven by energy optimization solutions, automation, and improved operational coordination. EEPQ - Vinamilk enables the company to better control electricity consumption, reduce unnecessary cold storage costs, and enhance profit efficiency for each hour of actual operation. The parameters used in the model, sourced from Vinamilk's financial reports, annual reports, and industry standards, ensure high accuracy and practical applicability, particularly in the context of a fiercely competitive market and growing demands for green production. These results not only deliver economic benefits but also bring Vinamilk closer to its sustainable development goals. By reducing energy consumption and optimizing production processes, EEPQ - Vinamilk helps the company lower greenhouse gas emissions, meeting increasingly stringent international environmental standards. This is particularly critical in the dairy industry, where energy costs for cold storage and production line operations account for a significant portion of expenses. The model also strengthens Vinamilk's competitive position in the international market, as partners and customers increasingly prioritize brands committed to sustainability. Furthermore, EEPQ - Vinamilk offers valuable lessons for small and medium-sized enterprises (SMEs) in the food industry, demonstrating that significant production efficiency improvements are possible without large investments in new infrastructure, through intelligent management and energy integration.

To maximize the potential of EEPQ_S and EEPQ - Vinamilk, businesses must implement synchronized strategies: investing in energy-saving technologies (such as high-efficiency motors and smart sensors), automating production lines, and enhancing employee awareness of energy management and equipment maintenance. For example, Vinamilk's optimization strategies for the EEPQ_S model include: Investing in energy-saving technologies and production automation forms the foundation for Vinamilk to optimize the EEPQ_S model, enhancing economic efficiency and sustainability. Adopting advanced technologies like high-efficiency motors can reduce electricity consumption in dairy production processes, from processing to packaging. LED lighting systems in factories and cold storage facilities save energy for illumination while reducing maintenance costs due to their long lifespan. Additionally, integrating smart IoT sensors enables real-time monitoring and adjustment of energy consumption, minimizing waste in processes like cooling or packaging. ERP software integrated with EEPQ_S supports production planning based on actual demand, avoiding excess inventory and reducing storage costs, particularly critical in the dairy industry with stringent preservation requirements. Precise temperature control and supply chain integration are key to ensuring product quality and reducing energy costs. Vinamilk should install automated temperature monitoring systems with sensors for temperature and humidity in cold storage, maintaining optimal conditions (4°C for fresh milk) to minimize unnecessary energy consumption. Inverter-based cooling technology can save 15%–25% energy by adjusting cooling capacity to actual needs, while optimizing preservation processes with an optimal batch size ($P^*=$ 2,600 units) reduces cold storage door-opening frequency and heat loss, lowering energy costs and product spoilage rates. For the supply chain, Vinamilk should leverage market data for accurate demand forecasting, synchronizing delivery schedules with raw material suppliers (e.g., fresh milk, packaging) to reduce raw material storage time. Optimizing cold transport with energy-efficient vehicles and efficient routes ensures product quality, reduces energy costs, and reinforces sustainability in Vinamilk's supply chain. Training employees, developing decision-support software, and committing to sustainable development will enable Vinamilk to fully harness EEPQ_S's potential. Regular training programs on energy management, high-efficiency equipment operation, and quality control will help employees minimize errors in sensitive processes like packaging and cold storage, achieving low defect costs. Programs encouraging energy-saving initiatives will motivate employees to propose improvements, such as optimizing production schedules. Vinamilk should invest in EEPQ_S simulation software or AI applications to analyze production and energy data in real-time, supporting parameter adjustments like P^* and t_1^* to maximize profit (1,213,693 USD/hour as in EEPQ - Vinamilk). Most importantly, adopting EEPQ_S helps Vinamilk reduce CO2 emissions (0.5–0.7 tons per unit of energy saved), meeting international environmental standards and enhancing brand reputation. By combining these strategies, Vinamilk not only optimizes profit but also solidifies its leadership in the dairy industry, contributing to a green, competitive future in the global market.

In the long term, future research directions could focus on identifying sensitive parameter thresholds (e.g., energy cost E or electricity consumption W), expanding the model to multi-product supply chains, and developing decision-support software to enhance practical applicability. In summary, EEPQ_S and EEPQ - Vinamilk represent the future of production management: sustainable, efficient, and competitive, delivering not only economic benefits but also contributing to a greener industry, meeting global market demands, and protecting the environment for future generations. Implementing the EEPQ_S model with the above synchronized strategies will enable Vinamilk to optimize profit (up to 1,213,693 USD/hour as in EEPQ - Vinamilk), minimize energy consumption, control costs, and enhance product quality. From investing in energy-saving technologies and production automation to training employees and integrating supply chains, Vinamilk can strengthen its leadership in the dairy industry while actively contributing to sustainable development and environmental protection goals. These solutions not only provide economic benefits but also create long-term value, helping Vinamilk maintain a competitive edge in domestic and international markets.

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