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ĐỒ ÁN TỐT NGHIỆP

Tên đề tài:

ENHANCING ENERGY EFFICIENCY IN SUPPLY CHAINS CONSIDERING PRODUCT DETERIORATION

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ABSTRACT

This thesis focuses on optimizing inventory systems for perishable goods by formulating and analyzing two advanced Economic Production Quantity (EPQ) models under constraints related to product deterioration and energy consumption. The increasing emphasis on sustainable operations and energy-efficient practices, especially in cold supply chains, has highlighted the need for inventory models that capture both product quality degradation and energy-related costs. However, existing literature predominantly considers deterioration in isolation, while the role of energy – an increasingly significant cost component – remains largely overlooked.

To address this gap, the first model in this study assumes a constant deterioration rate, representing traditional approaches to inventory planning. In contrast, the second model introduces a variable deterioration rate governed by the Weibull distribution, which better reflects the time-dependent spoilage behavior of real-world perishable products. Additionally, the second model explicitly incorporates energy consumption in both the production and storage phases. These models are formulated as nonlinear cost minimization problems that capture complex interactions between production rate, inventory levels, energy use, and spoilage. They are solved using the Differential Evolution (DE) algorithm, which is well-suited for handling highly nonlinear and non-convex optimization problems.

Extensive numerical experiments were conducted to evaluate the effectiveness of both models under varying operational parameters, including demand rate, energy price, setup cost, and storage conditions. A real-world case study based on a frozen pork production and cold storage facility in Da Nang, Vietnam, is used to validate the applicability and performance of the proposed models. The results demonstrate that the model with variable deterioration and energy integration achieves a 13.25% reduction in total system cost compared to the baseline model with constant deterioration. Moreover, it completely eliminates spoilage-related losses and significantly improves production stability, especially under volatile energy pricing and uncertain demand environments.

This thesis contributes to both theoretical advancement and practical application by proposing a data-driven decision framework to guide model selection based on operational conditions. It also offers actionable recommendations for integrating energy considerations into inventory decision-making, thereby supporting broader goals in green supply chain management.

THE UNIVERSITY OF DANANG
UNIVERSITY OF SCIENCE AND TECHNOLOGY
FACULTY OF PROJECT MANAGEMENT

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   Chapter 2: Literature review and theoretical background

   Chapter 3: Economic production quantity model (EPQ) with constant deterioration and energy consideration

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Although this thesis may still contain certain limitations, I sincerely hope it marks a meaningful step in my academic journey and serves as a foundation for further development in my future career.

Da Nang, June 2025

Student

Nguyen Le Cam Tu

DECLARATION

I hereby declare that the graduation thesis entitled **"Enhancing energy efficiency in supply chains considering product deterioration"** is the result of my own independent work, carried out under the supervision of Dr. Nguyen Hong Nguyen.

All data, results, and content presented in this thesis are original and have not been copied from any other source without proper citation. The thesis has not been submitted for any other degree or qualification at any other institution. All sources of information and materials used have been clearly referenced.

I take full responsibility for the integrity and authenticity of this work in accordance with the regulations of the university.

Da Nang, June 2025
Student

Nguyen Le Cam Tu

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LIST OF ABBREVIATIONS

Abbreviation	Full Term
GHG	Greenhouse gas
EOQ	Economic order quantity
EPQ	Economic production quantity
EEPP	Energy-efficient production planning
JELS	Joint economic lot sizing
EEPQ-D	Energy-based epq model with constant deterioration rate
EEPQ-V	Energy-based epq model with variable deterioration rate
ATC	Average total cost
SC	Setup cost
DC	Deterioration cost
EC	Energy cost
HC	Holding cost
BC	Backorder cost
DE	Differential evolution (optimization algorithm)

CHAPTER 1 INTRODUCTION

In this chapter, the thesis introduces the research context and highlights the necessity of the proposed model, particularly in an increasingly sustainability-driven business environment. The chapter begins by describing the current state of supply chains, with a focus on the challenges of managing perishable goods amid the ongoing energy crisis. It then analyzes the issue of energy consumption in production and storage activities, identifying research gaps in conventional inventory models. Based on this foundation, the chapter defines the research objectives and key research questions. Finally, an overview of the thesis structure is presented, clarifying the logical connections between chapters and the core content developed throughout the study.

1.1 Global context and research motivation

The ongoing global energy crisis, largely fueled by geopolitical instability, has underscored the urgent need to optimize energy consumption and minimize the environmental impact of supply chains.

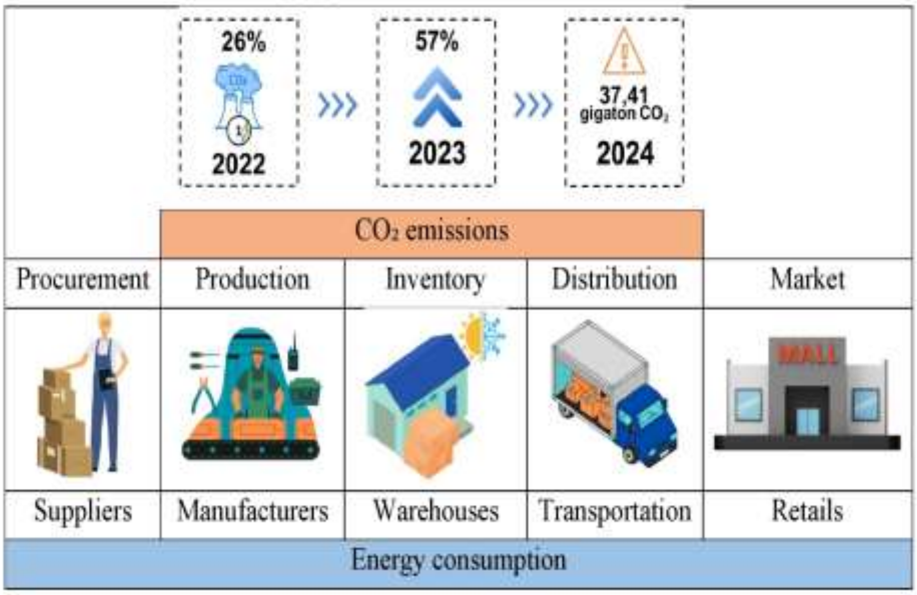

Figure 1.1 Energy consumption and gas emission in supply chain (recent year)

In 2022, the industrial sector was responsible for approximately 26% of global CO_2 emissions, amounting to around 9 gigatons of CO_2 [1], and remained the largest energy-consuming sector worldwide In 2023, global energy consumption increased by 2.2%, with the manufacturing and construction industries together accounting for 57% of total emissions [2]. Projections for 2024 estimate that CO_2 emissions from fossil fuel use and industrial activities will reach a record 37.41 gigatons [3] (see Figure 1.1). Despite efforts to mitigate climate change, post-pandemic industrial recovery has contributed to a disproportionate rise in energy use – driving 75% of the global energy demand growth since COVID-19. Moreover, the industrial sector is expected to contribute nearly two-thirds of the projected 2.5% increase in global natural gas demand in 2024 [4].

Within this broader context, refrigeration – a critical element for preserving temperature sensitive goods alone contributes to nearly 30% of industrial energy usage [5]. Furthermore, product deterioration exacerbates resource waste and environmental pollution. Global municipal solid waste is projected to increase by 70%, rising from 2.01 billion tons in 2016 to 3.4 billion tons by 2050 [6]. Improper disposal of deteriorated goods releases hazardous substances, endangering ecosystems and public health, while inefficient resource use further amplifies greenhouse gas emissions.

Moreover, global industrial energy consumption is projected to rise significantly by 2050, as illustrated in Figure 1.2, underscoring the urgent need for more energy-efficient practices [7]. This trend emphasizes the imperative for industry to transition towards more energy-efficient and sustainable production practices.

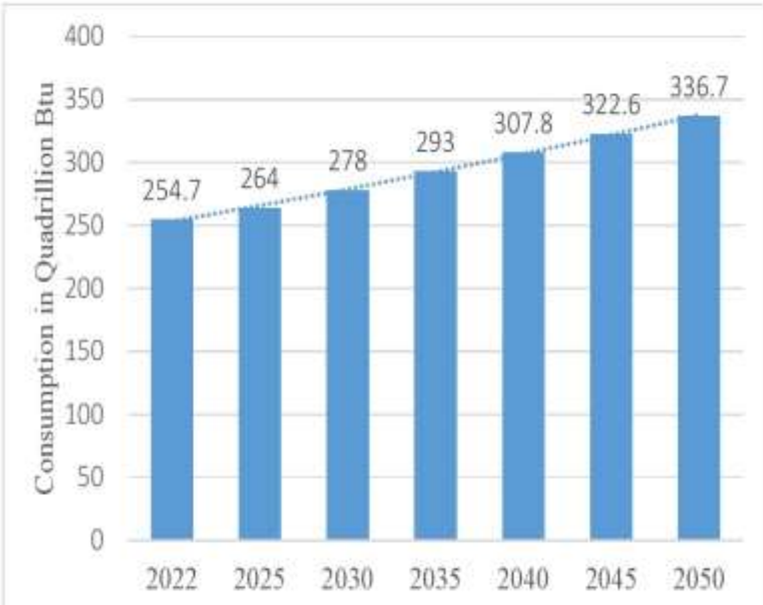

Figure 1.2 Industrial energy consumption worldwide in 2022, with a forecast until 2050, by energy source (Statista,2025)

In light of the projected surge in industrial energy demand, businesses must recognize that energy consumption, particularly in production and transportation stages of the supply chain is becoming increasingly subject to environmental regulations and energy efficiency policies. In this context, optimizing energy usage is not only a strategy for reducing operational costs but also a critical requirement for adapting to evolving environmental standards. More importantly, it represents a strategic step toward achieving sustainable production, minimizing carbon footprint, and improving resource efficiency. To address these challenges, it is essential for enterprises to explore and adopt energy-efficient management models within manufacturing environments, which will be further discussed in the next subsections 1.2.

1.2 Perishable products and energy-efficient in manufacturing

1.2.1 Characteristics of perishable products

Perishable products are items whose physical, chemical, or biological properties deteriorate over time, resulting in reduced quality, usability, or safety. This category includes fresh food, dairy products, seafood, pharmaceuticals, and biological materials all of which share a common feature: a short shelf life and high sensitivity to storage conditions. As a result, their production, transportation, and storage require strict environmental controls, including temperature, humidity, and holding time.

One of the major challenges in managing perishable goods is their high deterioration rate, which not only diminishes product value but also leads to resource waste, increased disposal costs, and environmental harm. Improper handling or disposal of spoiled goods can result in pollution, leakage of hazardous substances, and significant risks to public health.

Figure 1.3 Illustration of product deterioration over time

Recent studies have modeled the deterioration behavior of perishable products using various probability distributions, including Weibull, Triangular, Uniform, and Exponential Decay. Among these, the Weibull distribution is the most widely adopted due to its flexibility in capturing different degradation patterns over time [8]. In this study, the Weibull distribution is used to represent product deterioration in the inventory optimization model. Due to their sensitivity to time and environmental conditions, perishable goods are also among the most energy-intensive product categories, particularly in cooling systems and low-temperature production lines. It is estimated that refrigeration for perishable items accounts for nearly 30% of total energy usage in industrial, and this share is expected to rise as cold storage capacity expands in response to growing demand.

Therefore, optimizing energy consumption in the manufacturing of perishable goods is not only an economic necessity but also a strategic requirement for sustainable development. The following section explores approaches for improving energy efficiency in manufacturing environments and identifies suitable strategies for industries dealing with time-sensitive and perishable products.

1.2.2 Energy-efficient planning and inventory model

To improve energy efficiency in manufacturing systems, two main approaches have been highlighted in the literature: (i) technological innovations in production equipment, and (ii) managerial strategies that incorporate energy-related objectives into production planning and scheduling. While technological improvements often require significant capital investment, managerial models, particularly those under the umbrella of energy-efficient production planning (EEPP) have received increased attention due to their lower implementation costs and operational flexibility. EEPP models extend the objectives of traditional production planning by incorporating energy-related factors into cost functions or operational constraints. In addition to minimizing conventional costs such as setup and inventory holding, these models aim to reduce energy consumption, peak power demand, and associated greenhouse gas (GHG) emissions. This integration allows firms to optimize their operations not only economically, but also in line with environmental regulations and sustainability goals.

Among the various decision problems within EEPP, *lot sizing* plays a particularly critical role in balancing cost efficiency and environmental impact. Traditionally, lot sizing models have focused on minimizing the trade-off between setup costs and inventory holding costs. However, recent research has expanded these models to account for energy consumption during setup and production, recognizing the significant contribution of manufacturing operations to industrial energy use. For example, Collier and Ornek [9] introduced one of the first lot sizing models that incorporated energy costs in multi-stage production systems. Building upon this, Zanoni et al. [10] developed a model that accounts for different machine operating modes active, idle, and switching and demonstrated their impact on total energy usage. Other studies, such as those by Özdamar and Birbil [11] and Bazan et al. [12], further extended the modeling framework by integrating carbon emissions and environmental taxation policies into production decisions.

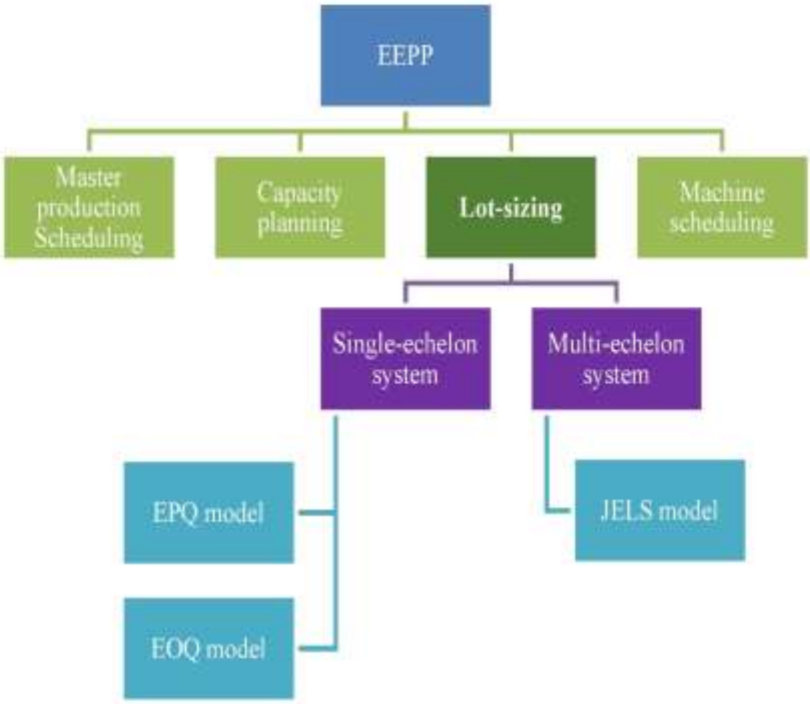

Figure 1.4 Classification of Energy-Efficient Production Planning Models

Among classical inventory models, the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models are foundational frameworks widely used for optimizing lot sizing. While EOQ assumes external procurement and instantaneous replenishment, EPQ is specifically designed for internal production settings, where production and consumption occur simultaneously. Given the focus of this research on energy-intensive, in-house production systems involving perishable products, *the EPQ model is adopted as the core modeling framework and further extended to incorporate energy consumption and product deterioration.* This modeling choice will be elaborated and justified through a comparative analysis of EOQ and EPQ in the Literature review chapter.

1.3 Research objectives

This research aims to develop an extended Economic Production Quantity (EPQ) model tailored for perishable products, by integrating key real-world factors that are often overlooked in classical inventory models. These include product deterioration over time, energy consumption in production and storage, and the possibility of backordering when inventory shortages occur.

The core objective is to construct a model that not only enhances cost-efficiency but also contributes to sustainable production planning, especially in energy-intensive manufacturing environments such as cold chains.

To achieve this, the study sets out the following specific objectives:

(1) *Develop a comprehensive EPQ-based inventory model that incorporates*: Traditional cost components (setup and holding costs), Energy consumption in both the production and warehousing phases, Time-dependent product deterioration (modeled using appropriate distributions Weibull), Backordering costs.

(2) *Evaluate the proposed model* through numerical analysis and comparison with traditional EPQ models for deterioration, to assess its advantages in terms of total cost reduction and operational practicality.

(3) *Perform sensitivity analysis* to identify how key parameters – such as deterioration rate, energy intensity, and setup frequency affect total cost and inventory decisions, thereby providing actionable insights for managers in perishable goods industries.

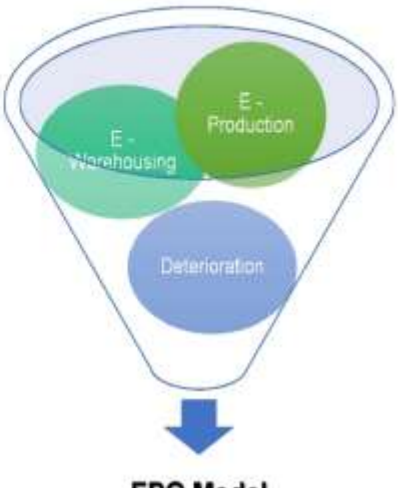

Figure 1.5 The objective model with energy implication for deterioration

Based on these objectives, the study addresses the following research questions:

(1) How can product deterioration, particularly for continuously decaying items, be effectively integrated into an inventory model?

(2) How can energy consumption during production and storage be modeled and optimized within inventory decisions for perishable items?

(3) What is the impact of deterioration, energy use, and setup frequency on the total cost structure and operational performance of a production-inventory system?

1.4 Structure of the Thesis

This thesis is structured into six main chapters, each contributing to a comprehensive investigation of inventory optimization for perishable products under energy constraints.

Chapter 1 – Introduction: This chapter outlines the research background, motivation, and objectives. It introduces the significance of integrating energy considerations into inventory management for perishable products.

Chapter 2 – Literature review and theoretical background: This chapter provides a comprehensive review of existing studies related to inventory models for deteriorating items and those incorporating energy consumption. In addition, it introduces fundamental concepts and modeling components that serve as the foundation for the development of the proposed framework. It identifies key research gaps that this thesis aims to address.

Chapter 3 – Economic production quantity model with constant deterioration and energy consideration: This chapter presents the baseline inventory model, which assumes a constant deterioration rate and includes energy consumption costs in both production and storage stages. The mathematical formulation and optimization approach are discussed in detail.

Chapter 4 – Extended EPQ model incorporating variable deterioration rates and energy efficiency considerations: This chapter extends the baseline model by allowing deterioration to follow a time-dependent Weibull distribution. It analyzes how spoilage dynamics interact with energy factors and compares model behavior with the constant-rate formulation.

Chapter 5 – Case Study and Managerial Implications: This chapter applies both models to a real-world frozen pork supply chain in Da Nang, Vietnam. It presents simulation results, compares performance under varying deterioration conditions.

Chapter 6 – Conclusion and Future Research Directions: The final chapter summarizes the main findings, discusses theoretical and practical contributions, and outlines potential directions for future research.

CHAPTER 2 LITERATURE REVIEW AND THEORETICAL BACKGROUND

This chapter presents a comprehensive review of existing literature related to inventory models for perishable products, with a particular focus on models that integrate energy consumption and carbon emissions in production and storage operations. By synthesizing prior research, this chapter highlights the current advancements in the field while identifying critical research gaps, especially in the context of sustainable and energy-efficient supply chains.

In addition to reviewing the literature, this chapter introduces key theoretical foundations that support the development of the proposed model. These include the characteristics of deteriorating items, the Weibull distribution for modeling time-dependent deterioration, the Specific Energy Consumption (SEC) function. Together, these elements form the scientific and analytical basis for the modeling approach elaborated in the following chapters.

2.1 Fundamental inventory models

In this section, we begin by revisiting the fundamental concepts of the Economic Order Quantity (EOQ) model, which serves as a foundational theory in inventory management. We then transition to the Economic Production Quantity (EPQ) model, which is more suitable for production environments where items are manufactured internally rather than procured. By examining the structure and assumptions of both models, we aim to identify the more appropriate framework for inventory control in production settings, particularly those involving perishable items and energy-related implication.

2.1.1 Economic order quantity model (EOQ)

The Economic order quantity model (EOQ) is a foundational concept in inventory management that determines the optimal order quantity a company should purchase to minimize the total costs associated with inventory. These costs include ordering costs (associated with placing orders) and holding costs (incurred from storing inventory). Introduced by Ford W. Harris in 1913 [13], the EOQ model remains widely applicable due to its simplicity and effectiveness in balancing trade-offs in inventory decisions. To provide a clearer understanding of the Economic Order Quantity (EOQ) model, Figure 2.1 illustrates the fundamental behavior of inventory levels over time under the EOQ assumptions.

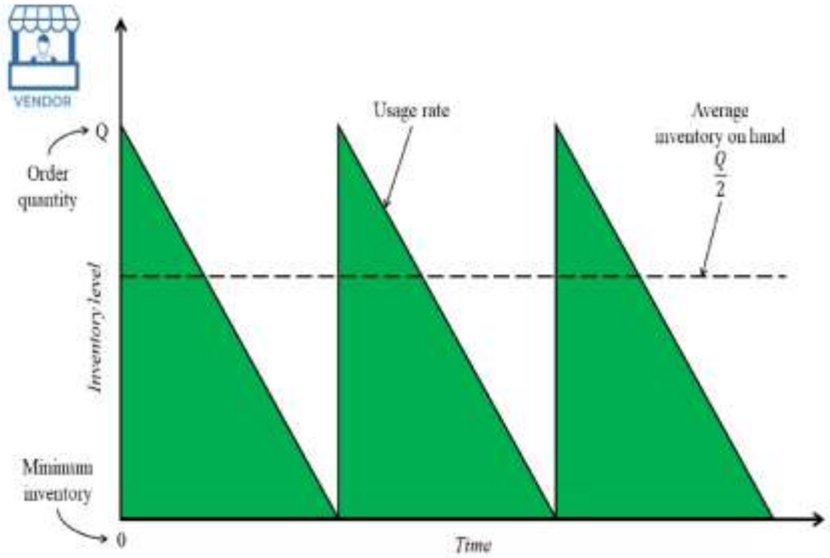

Figure 2.1 Economic Order Quantity (EOQ) Model Chart

The classical EOQ model assumes a constant and known demand rate, instantaneous replenishment, and no shortages allowed. Under these assumptions, the EOQ formula is derived to find the optimal order quantity that minimizes the total cost per cycle. The formula is expressed as:

$$EOQ = Q_o = \sqrt{\frac{2DS}{H}} \tag{1}$$

The annual total cost in the EOQ model consists of two components: ordering cost and holding cost. The total cost function is expressed as follows:

$$\begin{aligned} TC(Q_o) &= Ordering\ cost + Holding\ cost \\ &= \frac{DS}{Q} + \frac{HQ}{2} \end{aligned} \tag{2}$$

Where:

- D : Demand, usually in units per year
- S : Ordering cost per order
- H : Holding (carrying) cost per unit per year
- Q : Economic order quantity (EOQ)

The EOQ model provides several benefits, including reducing stockouts, avoiding overstock, and improving cash flow. However, its applicability is limited in complex environments where factors such as product deterioration, variable demand, energy consumption, or backordering must be considered.

Over time, the classical EOQ model has been significantly extended to better capture the complexities of real-world inventory systems. Researchers have incorporated practical factors such as backordering, stock shortages, variable holding costs, time-varying demand, energy consumption, and environmental sustainability [14 -17]. These advancements have greatly enhanced the model's applicability in modern supply chain contexts. In particular, they have laid a strong foundation for the development of more advanced models that are more suitable for manufacturing environments, such as the Economic Production Quantity (EPQ) model, which incorporates internal production processes and will be discussed in the following section.

2.1.2 Economic production quantity Model (EPQ)

The economic production quantity model (EPQ) is an extension of the EOQ framework that is tailored for situations where products are produced internally rather than procured from external suppliers. Unlike the EOQ model, which assumes instantaneous replenishment, the EPQ model accounts for a finite and continuous production rate, allowing inventory to gradually accumulate while demand is simultaneously fulfilled. In this model, production occurs at a constant rate P, which is assumed to be greater than the constant demand rate D. As production progresses, inventory increases at a net rate of $P - D$ and reaches a maximum before being depleted during the non-production period due to ongoing customer demand. This cyclical pattern reflects real conditions in many manufacturing systems.

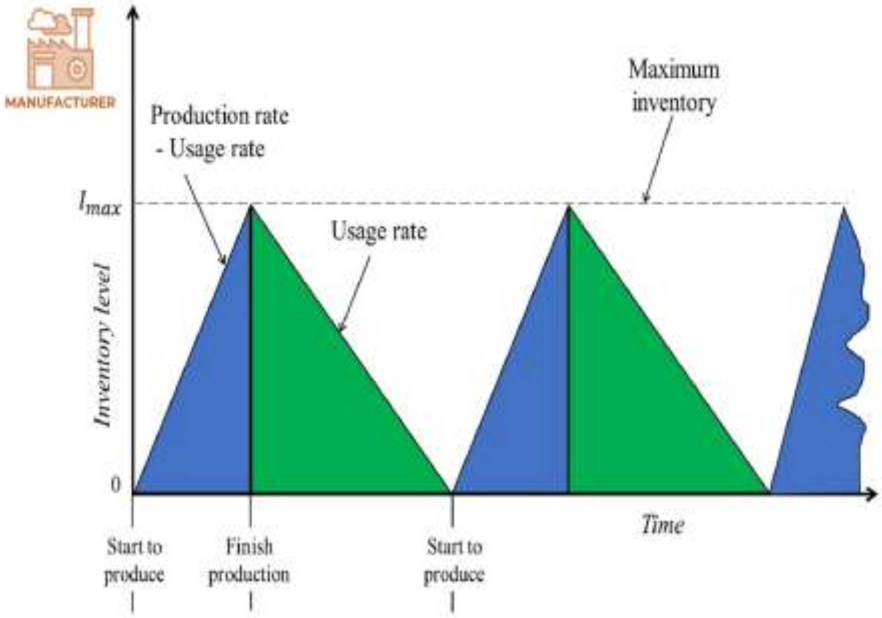

Figure 2.2 Economic Production Quantity (EPQ) Model Chart

The optimal production lot size in the EPQ model is given by:

$$EPQ = Q_p = \sqrt{\frac{2DS}{H}\frac{p}{p-d}} \quad (3)$$

The total annual cost in the classical EPQ model includes the setup cost and the holding cost. The total cost function is expressed as:

$$\begin{aligned} TC(Q) &= Setup\ cost + Holding\ cost \\ &= \frac{DS}{Q} + \frac{HQ}{2}\left(1 - \frac{p}{u}\right) \end{aligned} \quad (4)$$

Where:

- D : Demand, usually in units per year
- S : Ordering cost per order
- H : Holding (carrying) cost per unit per year
- Q : Economic production quantity
- p : Production or delivery rate
- d : Usage rate

This cost function reflects the fact that, due to simultaneous production and consumption, the average inventory level is lower compared to the EOQ model. Therefore, the EPQ model provides a more accurate and cost-effective inventory strategy for in-house production systems. In recent years, the EPQ model has been further enhanced to incorporate more realistic conditions, such as product deterioration, shortage allowances, energy consumption, carbon emissions, and sustainability concerns. These extensions improve the model's applicability to modern manufacturing systems, particularly in industries that deal with perishable goods or high energy demands.

2.2 Inventory models with deteriorating items

To examine deterioration aspects, this section focuses on two parts: (1) concepts and characteristics of deterioration, and (2) inventory management for deteriorating items using the Weibull distribution.

2.2.1 Concepts and characteristics

Inventory models that consider product deterioration have been developed to reflect the reality that many goods lose value, usability, or quality over time. This issue is particularly critical in industries such as food processing, pharmaceuticals, and chemicals, where quality degradation during storage can significantly increase operational costs and environmental burden. Traditional EOQ and EPQ models often assume perfect, non-deteriorating items, which simplifies the analysis but may result in overstocking, product waste, and suboptimal financial performance.

Deterioration is defined as the reduction in quantity or quality of inventory during the storage period due to spoilage, expiration, damage, or obsolescence. These losses directly affect stock levels and cost structures, making deterioration a critical factor in supply chain decision-making. Key characteristics of deterioration-based models include:

- Time-dependent decay, requiring inventory levels to be adjusted over the planning horizon.
- Increased cost and risk associated with large inventory volumes, leading to a preference for smaller and more frequent replenishments.
- Greater importance of storage conditions, as environmental factors may accelerate or slow down product degradation.
- Sustainability implications, as improper handling of deteriorated goods contributes to resource waste and environmental pollution.

Numerous studies have incorporated deterioration into inventory models using different assumptions. For example, Sani[18] and Daryanto & Wee[19] assumed a constant deterioration rate, which simplifies the calculations but fails to capture nonlinear decay behavior observed in practice. In contrast, more advanced models by Khalilpourazari et al.[20] and Palanivel & Uthayakumar[21] allow for variable deterioration rates, enhancing the accuracy of inventory decay predictions.

Beyond deterioration, inventory shortages are another critical challenge in modern supply chain management. Integrated models, such as by Pal et al.[22], have improved the balance between stock levels and ordering costs by accounting for both deterioration and backorders. However, even these advanced models often neglect energy consumption during production and storage, a factor that is becoming increasingly important in the context of sustainable operations. As recent data indicate, the industrial sector's share of total energy consumption is expected to continue rising through 2050. In response, governments have introduced stricter energy pricing and carbon reduction policies targeting not only domestic but also global enterprises. This trend highlights the urgent need for inventory models that integrate energy consumption as a core component of cost and sustainability analysis.

2.2.2 Deteriorating inventory management and Weibull distribution

To accurately model inventory deterioration over time, many researchers have adopted the Weibull distribution, a flexible probability function capable of representing various deterioration behaviors. Unlike models that assume a constant deterioration rate, the Weibull distribution allows for increasing, decreasing, or constant rates of deterioration depending on product characteristics. This flexibility enhances its applicability to industries dealing with time- or temperature-sensitive goods. In their study on inventory models for perishable items, Convert and Philip [23] emphasized that the Weibull distribution is particularly suitable for capturing product deterioration. In single-echelon systems, the deterioration rate is typically expressed using a two-parameter Weibull function.

$$L(t) = \alpha\beta t^{\beta-1} \tag{5}$$

Where

$L(t)$: Deterioration rate in period t

α : Scale parameter

β : Shape parameter

Depending on the value of β, the distribution behaves as follows:

$\beta < 1$: decreasing deterioration

$\beta = 1$: constant deterioration (equivalent to exponential decay)

$\beta > 1$: increasing deterioration

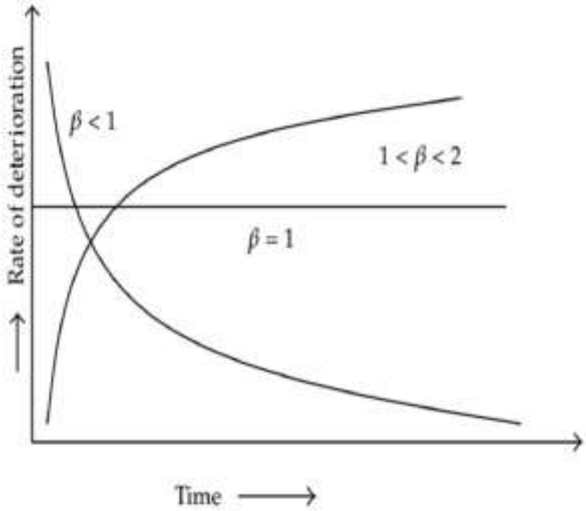

Figure 2.3 Deterioration rate over time under Weibull distribution

Owing to its modeling versatility, the Weibull distribution has emerged as the most prevalent approach in deterioration-based inventory models, as evidenced by the works of Mishra [24], Misra [25] and Wee [26]. Its ability to accurately characterize varying deterioration patterns empowers decision-makers to determine optimal replenishment cycles, design appropriate storage conditions, and develop effective production schedules in alignment with product perishability. Beyond operational planning, the incorporation of the Weibull distribution into inventory frameworks facilitates the formulation of comprehensive cost models that holistically account for setup costs, holding costs, shortage penalties, spoilage-induced losses, and energy expenditures. The integration of deterioration dynamics with energy considerations offers a more realistic and system-oriented representation of cost behavior within inventory systems.

These integrated models not only enable more precise cost minimization strategies but also contribute to improved operational sustainability. By capturing the interdependencies between perishability and energy usage, they support the transition toward environmentally conscious production systems, aligning closely with the broader objectives of sustainable industrial development.

2.3 Energy consumption in production and storage

In modern production systems, energy is not merely a cost component but a strategic variable that reflects environmental responsibility and regulatory compliance. As governments increasingly tighten carbon emission policies and energy prices continue to rise, developing inventory models that account for energy consumption has become a vital tool for balancing profitability with sustainable development. To support the integration of energy-related costs into optimal inventory models, recent studies have examined energy consumption in two distinct phases: production and warehousing. These efforts have led to the emergence of diverse and practical modeling approaches for industrial applications. A detailed analysis of energy use in both phases is essential to gain a deeper understanding of cost components and to lay the groundwork for constructing an optimized total cost function.

2.3.1 Energy consumption in production processes

Modern inventory models have increasingly incorporated energy costs into their overall cost structures. Energy consumption in the production phase has been studied from both technical and managerial perspectives. While traditional inventory models typically assume static cost parameters, real-world production costs are influenced not only by setup and labor but also by machine efficiency, idle time, and electricity intensity. Marchi et al. [27] and Ferretti et al. [28] point out that energy usage in production can account for a significant portion of total operating costs, especially in high-volume or high-load batch production systems. Ignoring this factor may lead to inaccurate cost evaluations. Zavanella et al. [27] and Nguyen et al. [29] introduced the concept of Specific Energy Consumption (SEC), which models energy consumption per unit of product as a function including constant, linear, or nonlinear of the production rate. This approach allows for more precise estimation of energy-related costs under varying operating conditions.

Some studies have further expanded these models by incorporating additional factors such as adjustable production rates, machine idle time, and carbon emission policies. However, most existing models still do not consider product deterioration, a critical element in actual manufacturing environments. The absence of this factor reduces the accuracy of cost and sustainability assessments. As highlighted by Nguyen et al. [30] integrating both energy consumption and deterioration provides a more accurate reflection of a product's lifecycle cost. Their findings suggest that optimal inventory policies must simultaneously balance production lot sizes, energy efficiency, and deterioration dynamics in order to minimize total costs while meeting long-term environmental objectives.

2.3.2 Energy consumption in warehousing

Warehousing, particularly in industries requiring temperature control such as cold food chains and pharmaceuticals, is a major source of energy consumption and CO_2 emissions. Activities such as refrigeration, lighting, ventilation, and equipment operation occur continuously, even when inventory levels are low. Studies by Marchi et al. [17] and Fichtinger et al. [31] emphasize that warehouse energy consumption should not be treated as a fixed cost. Instead, it should be modeled as a function of space utilization, insulation efficiency, ambient temperature difference, and storage condition requirements.

The concept of Specific Energy Consumption (SEC) has also been applied to warehousing, allowing energy use to be calculated per unit of stored product over time. Zanoni [32] extended this approach to a serial supply chain context, analyzing energy flows across multiple stages of storage and transportation. Their models incorporate factors such as effective storage temperature, safety stock levels, and storage duration making them highly relevant for perishable goods. These findings highlight the potential of applying SEC-based modeling to inventory systems handling deteriorating products. Tiwari et al. [33] and Daryanto & Wee [19] linked warehouse energy use with CO_2 emissions in models for deteriorating items. Their inventory frameworks aimed to simultaneously minimize economic costs and environmental impact by including variables such as product footprint, refrigeration load, and shipment frequency. However, despite these advancements, most models still do not fully integrate product deterioration, even though real-world product quality can deteriorate rapidly under poor storage conditions, leading to financial losses and increased environmental burden.

Moreover, models that focus solely on energy or emissions often overlook the indirect impact of inventory shortages on energy consumption. In scenarios where customers are willing to wait for backordered items, firms can reduce production and storage intensity during certain periods, thereby lowering energy use. This observation suggests that allowing for backorders can support energy-saving strategies when managed properly. Therefore, a truly sustainable inventory model must integrate energy consumption, product deterioration, and shortage behavior simultaneously. As emphasized by Nguyen et al. [34], only by addressing these three dimensions holistically can businesses develop inventory strategies that are both economically efficient and aligned with long-term environmental objectives.

2.3.3 Specific Energy Consumption Function (SEC)

In the context of growing emphasis on sustainability and energy cost optimization, accurately incorporating energy consumption into inventory models has become essential. To address this need, this study adopts the concept of Specific Energy Consumption (SEC), which quantifies the energy consumed per unit of product as a function of operational parameters – rather than treating energy costs as static values, as is common in traditional models. Formulating the SEC function for both production and warehousing stages provides a comprehensive foundation for evaluating energy-related costs in inventory systems, especially those involving deteriorating products.

In the production phase, the SEC function is often modeled in a nonlinear form to capture both idle energy consumption and additional energy usage during active production. A typical SEC function proposed by Nguyen et al [29] is formulated as:

$$SEC = \frac{W}{P} + K \quad (6)$$

Where:

- W : Idle power consumption (kW)
- P : Production rate (units/hour)
- K : Energy required to produce one unit of product (kWh/unit)

This function illustrates that energy consumption per unit decreases as the production rate increases, thus supporting optimal production lot sizing decisions aimed at enhancing energy efficiency.

In the warehousing phase, particularly in temperature-controlled storage for perishable products, the SEC function captures the relationship between storage occupancy, insulation performance, and the temperature differential between internal and external environments. According to the model by Marchi et al. (2020), the SEC for refrigerated storage can be expressed as:

$$SEC(T_w, I_t) = SEC(T_r, I_t) I_{max} \rho \quad (7)$$

Where:

- T_w : Expected warehouse temperature (°C)

T_r : Referenced warehouse temperature (°C)

I_t : Inventory level at time t (unit)

I_{max} : Maximum storage capacity of the warehouse (unit)

ρ : Coefficient linking SEC to various storage temperatures

This function accurately reflects the impact of environmental conditions and inventory volume on warehouse energy costs. Thus, its application enables the total cost function to realistically capture electricity usage in storage and the efficiency of energy use based on inventory level and temperature control.

Integrating SEC functions into inventory models not only provides a realistic representation of energy dynamics within the system but also bridges the gap between economic performance and environmental responsibility. These functions offer a quantitative basis for optimizing both energy performance and inventory strategies, thereby supporting sustainable decision-making in production and supply chain operations. Through this approach, the study contributes to building a comprehensive modeling framework that tightly links energy consumption, product preservation quality, and overall cost optimization.

2.4 Research gap and contributions

Although a considerable body of literature exists on the Economic Production Quantity (EPQ) model, most studies tend to treat the complexities of modern production systems in isolation. Specifically, two crucial factors product deterioration and energy consumption are often addressed separately rather than within a unified optimization framework. This separation results in fragmented models that fail to capture the full spectrum of operational trade-offs encountered in energy-intensive environments managing perishable goods.

On one hand, many EPQ-related studies have incorporated deterioration effects to reflect the reality that product quality and quantity degrade over time. These works have progressively evolved from assuming constant deterioration rates to modeling time-dependent spoilage using distributions like Weibull. On the other hand, recent literature has started to examine the role of energy consumption in both production and storage stages, acknowledging the increasing pressure from energy pricing and carbon emission regulations. However, such studies frequently neglect the dynamic behavior of perishability, resulting in limited applicability to cold chains or food and pharmaceutical sectors where deterioration and energy usage are tightly coupled. Moreover, only a small number of studies have attempted to incorporate both deterioration and energy cost in the same model, and even fewer have done so with rigorous mathematical integration or real-world validation. Existing models also tend to focus on deterministic inputs without adequately capturing uncertainties in demand or spoilage behavior, further constraining their utility in practical decision-making scenarios.

Table 2.1 provides a comparative summary of prior works, highlighting the extent to which key factors such as deterioration modeling, energy costs, backordering, and sustainability objectives have been integrated. This analysis reveals a clear lack of holistic frameworks that simultaneously consider nonlinear deterioration, energy consumption in both production and warehousing, and their compounded impact on inventory cost structures.

To address these research gaps, this study offers the following key contributions:

(1) Proposing an EPQ model for deteriorating items that integrates deterioration and energy consumption via the Weibull distribution for sustainable inventory management.

(2) Analyzes the combined effects of energy use, setup cost, and deterioration on total cost under realistic production conditions

(3) Analyze the role of product deterioration characteristics in shaping and altering the total cost structure of the inventory system.

The proposed framework is visually outlined in Figure 2.4, serving as a conceptual bridge between theoretical modeling and operational implementation. These advancements contribute to a more robust, sustainable, and context-aware inventory management strategy for perishable goods in energy-constrained environments.

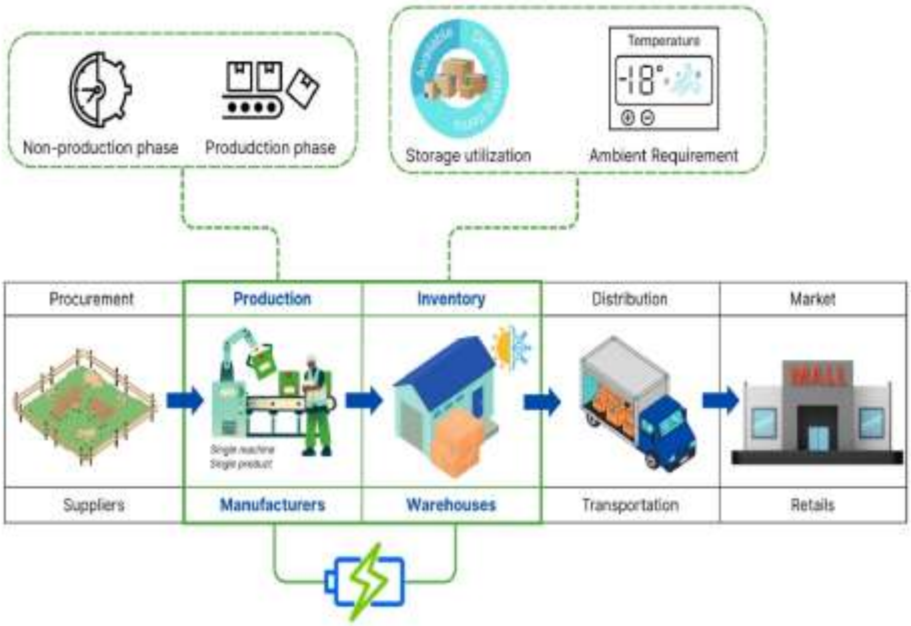

Figure 2.4 Production system with temperature-controlled storage and energy consumption

Table 2.1 Review of Integrated Inventory Models with Sustainability Considerations

Author	Production characteristic	Deterioration rate		Shortage	Energy implication		CO_2 implication
		Constant	Variable		Production	Warehousing	
Widyadana & Wee [35]	D	✓					
Zanoni et al. [32]	G				✓	✓	
Palanivel & Uthayakumar[21]	D		✓	✓			
Pal et al.[22]	D		✓	✓			
Mishra [24]	D		✓	✓			
Nezami & Heydar[36]	G				✓		
Palanivel & Uthayakumar[37]	D		✓				
Tiwari et al. [33]	D	✓					✓
Ferretti et al. [28]	G				✓		
Marchi et al. [38]	G				✓		
Daryanto & Wee [19]	D	✓					✓
Khalilpourazari et al. [20]	D		✓				
Marchi et al. [17]	C					✓	
H. N. Nguyen et al. [29]	G				✓		
Sani[18]	D	✓		✓			
Nguyen et al.[39]	G			✓	✓		
Nguyen et al.[30]	G				✓	✓	
Nguyen et al.[34]	G			✓	✓	✓	
This work	**D**	✓	✓	✓	✓	✓	

G: General; D: Deterioration;

CHAPTER 3 EPQ MODEL WITH CONSTANT DETERIORATION AND ENERGY CONSIDERATION

Based on the identified research gap and the theoretical foundations discussed in **Chapter 2**, **Chapter 3** develops an EPQ model that integrates energy consumption and product deterioration, in which the deterioration rate is assumed to be constant – a special case of the Weibull distribution. Following the model formulation, numerical analysis is conducted to demonstrate the model's practical relevance, and sensitivity analysis is performed to highlight the operational characteristics of the model, thereby providing relevant managerial insights.

3.1 Introduction and problem statement

This chapter presents an extended Economic Production Quantity (EPQ) model that integrates energy consumption and product deterioration into a unified decision-making framework for a single-product, single-machine production-inventory system as shown in Figure 3.1. The model is designed to reflect practical production environments where sustainability and energy efficiency are growing concerns, particularly for perishable goods. The system operates under a finite and adjustable production rate, which is always greater than the constant demand rate. Upon completion of a production run, the machine enters a standby mode, during which it continues to consume energy even while idle. This characteristic is important for realistically modeling total energy consumption over the entire production-inventory cycle.

To accurately model the degradation of product quality over time, the proposed framework adopts the Weibull distribution with shape parameter $\beta = 1$, which corresponds to an exponential decay pattern. This formulation preserves analytical tractability while still capturing essential features of time-dependent deterioration. Additionally, shortages are permitted in the form of backorders, providing inventory flexibility while enabling the model to reflect real-world supply chain conditions.

The model accounts for energy consumption in:

- During active production (rate-dependent energy usage),
- During non-production/standby periods (base-load energy consumption),
- During storage, where energy is required to maintain environmental conditions (e.g., refrigeration).

The primary objective is to minimize the total average system cost, which comprises the following components:

- Setup cost per cycle,

- Holding cost for inventory,
- Deterioration cost due to product decay,
- Shortage cost from backordered items,
- Energy cost in the production, standby, and storage phases.

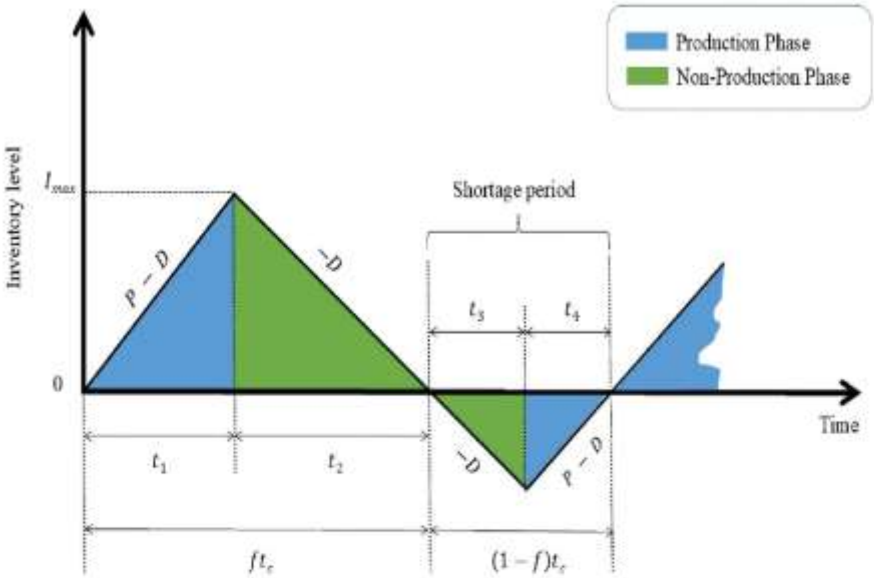

Figure 3.1 Inventory level over the production cycle

By jointly optimizing the cycle time, production rate, and machine utilization schedule, this model provides a comprehensive and sustainable approach to production-inventory planning that balances economic performance with energy and environmental considerations.

3.2 Notations and assumptions

3.2.1 Notations

The following notation is used to describe the model:

Parameters

D : Demand rate (unit/h)

S : Setup cost (\$/setup)

H : Holding cost per unit (\$/(unit.h))

C : Unit production cost (\$/(unit.h))

B : Backorder cost (\$/(unit.h))

W : Idle energy consumption (kW)

K : Energy consumption for one unit producing (kWh/unit)

E : Energy cost (\$/kWh)

$L(t)$: Deterioration rate, given by $\alpha\beta t^{\beta-1}$ where $\alpha, \beta, t > 0$. When $\beta = 1$, $L(t)$ becomes a constant which is case of an exponential decay.

T_w : Expected warehouse temperature (°C)

T_r : Referenced warehouse temperature (°C)

T_{hot} : Outside warehouse temperature (°C)

ρ : Coefficient linking SEC to various storage temperatures

λ, μ : Positive coefficients dependent on the characteristics of the warehouse, where $\mu \in (0, 1)$

δ, γ : Positive coefficients dependent on the filling level of the warehouse.

Dependent variables

t_1 : Time of production time (h)

t_2 : Time of non-production time (h)

t_3 : Time of consumption sub-time in shortage period (h)

t_4 : Time of production sub-time in shortage period (h)

I_t : Inventory level at time t (unit)

I_{max} : Maximum storage capacity of the warehouse (unit)

I_b : Stockout demand (units)

Decision variables

P : Production rate (unit/h)

t_c : Cycle time (h)

f : Fraction of period length with positive inventory level, $f \in (0, 1]$

3.2.2 Assumptions

The assumption of an inventory model for product life cycle are as follows:

- Demand is known and has a constant rate.
- The production rate is limited, greater than demand rate ($P > D$) and confine within a specific range [P_{min}, P_{max}].
- Changes to the production rate are implemented only at the beginning of the production cycle.
- Stortages are allowed with complete backlogging.
- Lead time is neglible.
- The units are available for satisfying demand immediately after their production.
- The deterioration rate is a constant rate, assuming that items begin to deteriorate immediately upon entering the inventory.
- Throughout each cycle, no replacement or repair of deteriorated items is performed.

- The cost of a deteriorated unit is constant and equal to the unit production cost C. This will account for the salvage value, if any.
- The machine remains in an idle state during the non-production period.

3.3 Mathematical model development

According to Figure 3.1, the inventory level starts at 0 when $t = 0$ and gradually increases to its maximum, I_{max}, at $t = t_1$. After t_1, production stops, and the inventory level decreases due to demand and product deterioration. At $t = t_2$, the inventory level drops back to 0. During t_3, customer demand exceeds supply, resulting in backorders that accumulate over time. Production resumes at t_4, during which the accumulated backorders are fulfilled alongside meeting new demand. Let $L(t)$ represent the deterioration rate fuction for items stocked, the inventory level of the system at time t over period $[0, t_c]$ can be described by the following equations.

$$\frac{dI_1}{dt} + L(t)I_1 = P - D, \quad 0 \leq t \leq t_1 \tag{8}$$

$$\frac{dI_2}{dt} + L(t)I_2 = -D, \quad 0 \leq t \leq t_2 \tag{9}$$

$$\frac{dI_3}{dt} = -D, \quad 0 \leq t \leq t_3 \tag{10}$$

$$\frac{dI_4}{dt} = P - D, \quad 0 \leq t \leq t_4 \tag{11}$$

The solutions to the above differential equations using Spiegel (1960) are as follows:

$$I_1 = \frac{Z_1 + \int_0^t (P - D)epx(\int L(t)dt)}{\exp(\int_0^t L(t)dt)}, \quad 0 \leq t \leq t_1 \tag{12}$$

$$I_2 = \frac{Z_2 + \int_0^t (-D)epx(\int L(t)dt)}{\exp(\int_0^t L(t)dt)}, \quad 0 \leq t \leq t_2 \tag{13}$$

$$I_3 = \int_0^t -Ddt, \quad 0 \leq t \leq t_3 \tag{14}$$

$$I_4 = \int_0^t (P - D)dt, \quad 0 \leq t \leq t_4 \tag{15}$$

The values of the constants of intergration Z_1, Z_2 can be found by using the boundary conditions. That is, at t=0, I_1=0, the initial inventory, and at $t = t_1$, $I_2 = I_{max}$; at $t = t_2$, $I_3 = 0$ and at $t = t_3$, $I_4 = I_b$, are

$$I_1 = \frac{\int_0^t (P - D)epx(\int L(t)dt)dt}{\exp(\int_0^t \int L(t)dt)}, \quad 0 \leq t \leq t_1 \tag{16}$$

$$I_2 = \frac{I_{max} + \int_0^t (-D)epx(\int L(t)dt)dt}{\exp(\int_0^t L(t)dt)}, \quad 0 \le t \le t_2 \tag{17}$$

$$I_3 = -Dt, \qquad 0 \le t \le t_3 \tag{18}$$

$$I_4 = (P - D)t, \quad 0 \le t \le t_4 \tag{19}$$

Now at $t = t_2$, $I_2 = 0$, hence

$$I_{max} = \int_0^{t_2} D\, exp\left(\int L(t)dt\right)dt \tag{20}$$

Substituting this in equa. (17) yields

$$I_2 = \frac{\int_0^{t_2} D\, exp(\int L(t)dt)dt + \int_0^t (-D)epx(\int L(t)dt)}{\exp\left(\int_0^t L(t)dt\right)} \tag{21}$$

In this case, we analysis $\beta = 1$, as previously established, the deterioration rate is constant and denoted by α. Accordingly, the expressions for I_1,I_0 , and I_2 can be systematically derived by substituting $\beta = 1$ into equations (16), (17), and (20), respectively.

$$I_1 \quad = \frac{\int_0^t (P - D)epx(\alpha t)dt}{\exp(\alpha t)}$$

$$= \frac{P - D}{\alpha}[1 - \exp(-\alpha t)] \tag{22}$$

Similarly, $t = t_1$or $t_2 = 0$, ,I_{max} can be write as:

$$I_{max} = \frac{P - D}{\alpha}[1 - \exp(-\alpha t_1)] \tag{23}$$

$$I_{max} = \int_0^{t_2} D\, exp(\alpha t)dt = \frac{D}{\alpha}[exp(\alpha t_2) - 1] \tag{24}$$

$$I_2 \quad = \frac{\int_0^{t_2} D\, exp(\alpha t)dt + \int_0^t (-D)epx(\alpha t)dt}{\exp(\alpha t)}$$

$$= \frac{D}{\alpha}\left[\frac{\exp(\alpha t_2) - \exp(\alpha t)}{\exp(\alpha t)}\right] \tag{25}$$

And from Fig. 3.1 the value of t_1, t_2, t_3, t_4 are caculated as

$$t_c = t_1 + t_2 + t_3 + t_4 = ft_c + (1 - f)t_c$$

From equations (23) and (24), and by expanding the exponential functions and disregarding second-order and higher-order terms of α for sufficiently small values of α, we obtain:

$$(P-D)\left[t_1-\frac{\alpha t_1^2}{2}\right]=D\left[t_2+\frac{\alpha t_2^2}{2}\right] \quad (26)$$

According to Misra [25], the Correction Method of Newton Jaffe et al. [40], previously applied by Covert & Philip [23] to a basic lot-sizing model with variable deterioration rates, can also be adopted effectively for simpler lot-sizing models involving variable deterioration rates. Nevertheless, obtaining exact solutions for the optimal values of the variables t_1 and t_2 directly from these equations is challenging. Therefore, an approximation technique is recommended to simplify the solution procedure. Specifically, the values of t_1 and t_2 approximately satisfy the following simplified equations:

$$\begin{cases} t_1-\dfrac{\alpha t_1^2}{2}=t_1 \\ t_2+\dfrac{\alpha t_2^2}{2}=t_2 \end{cases}$$

Substituting these approximations back into equation (26) yields:

$$t_1=\frac{D}{P}ft_c \quad (27)$$

$$t_2=\frac{P-D}{P}ft_c \quad (28)$$

From Fig. 3.1, it is apparent that $I_b=(P-D)t_4=Dt_3$ and $(t_3+t_4)=(t_c-t_1-t_2)$; Thus, the following expressions for t_3 and t_4 can be derived:

$$D(t_3+t_4)=Pt_4$$

$$t_4=\frac{D}{P}(1-f)t_c \quad (29)$$

$$t_3=\frac{P-D}{D}t_4=\frac{P-D}{P}(1-f)t_c \quad (30)$$

These dependent variables are used to construct the component costs, including setup cost, holding cost, deterioration cost, backorder cost, and energy consumption during both the production and storage phases. These components are then combined to formulate the total average cost function of the system.

3.3.1 Setup cost

In a production-inventory system operating under a cyclic policy, a setup cost is incurred each time a production cycle begins. This cost accounts for the preparation activities required to initiate production, including machine setup, material loading, and system initialization. Since the system follows a periodic replenishment strategy, the total setup cost is distributed over the cycle time.

Let:

S : be the setup cost per cycle (USD)

t_c : be the length of one production-inventory cycle (hours).

The average setup cost per unit time is then calculated by dividing the setup cost by the cycle time:

$$SC = \frac{S}{t_c} \tag{31}$$

This expression forms one component of the total average cost of the system and illustrates the trade-off between longer and shorter production cycles. A longer cycle reduces the frequency of setups, thereby lowering setup costs. However, it also requires storing more inventory, which can increase holding and deterioration costs. On the other hand, shorter cycles result in more frequent setups and higher setup costs, but may reduce inventory levels and the risk of product deterioration. Therefore, a proper balance must be achieved to minimize the overall system cost.

3.3.2 Holding cost

In inventory models with deteriorating items, holding cost reflects the expenses associated with storing products that gradually lose their value over time. Unlike classical models where holding cost is simply proportional to the average inventory level, models with deterioration must incorporate the degradation of stored goods into the cost function. In this system, the production-inventory cycle consists of two main phases:

- The production period t_1, during which inventory builds up.
- The non-production period t_2, during which inventory depletes due to demand and deterioration.

With I_1 and I_2 denote the inventory level during t_1 and t_2, respectively. The holding cost per unit time for deteriorating items in inventory models is formulated as follows:

$$\begin{aligned} HC &= \frac{H}{t_c}\left[\int_0^{t_1} I_1 dt + \int_0^{t_2} I_2 dt\right] \\ &= \frac{H}{t_c}\left[\int_0^{t_1} \frac{P-D}{\alpha}[1-\exp(-\alpha t)]dt + \int_0^{t_2} \frac{D}{\alpha}\frac{\exp(\alpha t_2)-\exp(\alpha t)}{\exp(\alpha t)}dt\right] \\ &= \frac{H}{t_c}\left[\frac{P-D}{2}\left(1-\frac{\alpha t_1}{3}\right)t_1^2 + \frac{D}{2}\left(1+\frac{\alpha t_2}{3}\right)t_2^2\right] \end{aligned}$$

Substituting t_1, t_2 as defined in equation (20), we obtain

$$HC \quad = \frac{HDf^2t_c(P-D)}{2P}\left(1 + ft_c\alpha\frac{(P-2D)}{3P}\right) \tag{32}$$

This expression reflects how holding cost is influenced not only by the cycle time and demand rate, but also by the production rate P, deterioration rate α, and the fraction f of the cycle during which inventory is available.

3.3.3 Deterioration cost

In inventory systems involving perishable items, deterioration cost accounts for the economic loss due to items degrading while held in stock. Since deterioration occurs continuously over time, the cost must reflect not only the inventory level but also the deterioration rate at each point in time. In this model, deterioration occurs in both the production period t_1 and the non-production (consumption) period t_2.

With $L(t) = \alpha$, denote the constant deterioration rate (as defined for $\beta = 1$ the Weibull distribution), and let C represent the cost per unit of deteriorated item. The deterioration cost per unit time is calculated by integrating the deteriorated quantity across the two time periods and multiplying by unit cost:

$$DC = \frac{C}{t_c}\left[\int_0^{t_1} L(t)I_1 dt + \int_0^{t_2} L(t)I_2 dt\right]$$

Since $L(t) = \alpha$ is constant, and using the expressions for I_1 and I_2, the deterioration cost becomes:

$$DC = \frac{C}{t_c}\left[\int_0^{t_1} \alpha\frac{P-D}{\alpha}[1-\exp(-\alpha t)\,dt] + \int_0^{t_2} \alpha\frac{D}{\alpha}\frac{\exp(\alpha t_2)-\exp(\alpha t)}{\exp(\alpha t)}dt\right]$$

$$= \frac{C}{t_c}\left[\frac{P-D}{2}\left(1-\frac{\alpha t_1}{3}\right)\alpha t_1^2 + \frac{D}{2}\left(1+\frac{\alpha t_2}{3}\right)\alpha t_2^2\right]$$

$$= \frac{\alpha f^2 t_c CD(P-D)}{2P}\left(1 + ft_c\alpha\frac{(P-2D)}{3P}\right) \tag{33}$$

This equation highlights how deterioration cost is jointly influenced by the production rate (P), demand rate (D), deterioration rate (α), cycle time t_c, and the f Fraction of period length with positive inventory level f. As expected, when $\alpha = 0$, the deterioration cost becomes zero, aligning with models of non-perishable goods.

3.3.4 Backorder cost

In this model, shortages are permitted and are fully backlogged. The backorder cost represents the penalty associated with not meeting demand immediately and delivering items later. Similar to the calculation of holding cost, the backorder cost is computed based on the amount of unfulfilled demand over the shortage period, which consists of two sub-intervals:

- t_3: time during which demand accumulates while no production occurs.
- t_4: time during which demand is met progressively as production resumes.

Let I_3 and I_4 denote the backordered quantity during t_3 and t_4, respectively. Then, the average backorder cost per unit time is given by:

$$BC = \frac{B}{t_c}\left[\int_0^{t_3} I_3 dt + \int_0^{t_4} I_4 dt\right]$$
$$= \frac{B}{t_c}\left[\int_0^{t_3} Dtdt + \int_0^{t_4} (P-D)tdt\right]$$
$$= \frac{B}{t_c}\left[\frac{-Dt_3^2}{2} + \frac{(P-D)t_4^2}{2}\right]$$
$$= \frac{B(1-f)^2 t_c}{2} D\left(1-\frac{D}{P}\right), \tag{34}$$

This expression shows that backorder cost increases with longer shortage durations, higher demand rates, and greater differences between production and demand. It decreases when the production rate is close to demand, or when the fraction of the cycle with inventory f is high.

3.3.5 Average related – production energy consumption cost

Energy consumption in the production-inventory system originates from two primary sources:

- During the active production phases t_1 and t_4, when the machine is manufacturing products.
- During the non-production phases t_2 and t_3, when the machine remains powered on but idle.

These two phases incur different forms of energy costs, which are captured using a Specific Energy Consumption (SEC) framework. The total average energy cost per unit time is the sum of the energy used during production and non-production periods.

During production periods, the machine consumes energy in two forms: a rate-dependent component and a fixed power draw. Specifically, each unit produced requires K(kWh/unit) of energy, while the machine also draws idle power W(kW) throughout the production period. When combined and expressed per unit of output, the average energy cost during production is given by:

$$EC_{prod.} = \left(\frac{W}{P} + K\right) DE \tag{35}$$

In contrast, during the non-production phases t_2 and t_3, the machine does not produce output but still consumes energy at a constant idle power W. Therefore, the average energy cost during non-production is:

$$EC_{nonp.} = \frac{WE}{t_c}(t_2 + t_3)$$
$$= WE\left(1 - \frac{D}{P}\right) \quad (36)$$

Summing both production and non-production phases yields the total average energy cost per unit time. This formulation helps quantify how decisions related to the production rate and cycle time influence overall energy efficiency, which is crucial in energy-sensitive and sustainability-oriented operations.

3.3.6 Average related – warehousing energy consumption cost

In addition to energy consumed during production and idle machine states, the system also incurs energy costs related to warehousing operations, particularly for maintaining environmental conditions such as refrigeration, ventilation, or temperature control. This type of energy usage is especially critical in systems storing perishable or temperature-sensitive items.

The warehousing energy consumption cost is computed based on the Specific Energy Consumption (SEC) model, which represents the amount of energy required to maintain one unit of product in the warehouse, as a function of both the storage temperature and the level of inventory in storage. The original SEC function was introduced by Zanoni et al. [32], where it was modeled as dependent on the inventory level. Later, Marchi et al.[17] extended this formulation by incorporating the influence of ambient temperature differences, making the model more suitable for cold supply chains and dynamic environments.

Accordingly, the average warehousing energy cost per unit time is defined as follows:

$$EC_{ware.} = \frac{E}{t_c}\int_0^{t_c} SEC(T_w I(t)) I_{max} dt \quad (37)$$
$$= \frac{E}{t_c}\int_0^{t_c}\left[\lambda I_{max}^{-\mu} + \delta\left(1 - \frac{I(t)}{I_{max}}\right)^{\gamma}\right]\rho I_{max} dt$$

The first component, $\lambda I_{max}^{-\mu}$, represents the basic SEC of the warehouse based on the volume reserved for product storage. Here, λ and μ are positive coefficients, with $\mu \in (0; 1)$. This component is influenced by ambient and required temperatures, the warehouse design, and operational methods. According to this function, the SEC value decreases as the maximum warehouse volume increases.

The second component, $\delta\left(1-\frac{I(t)}{I_{max}}\right)^{\gamma}$, captures the energy consumption arising from inefficiencies in warehouse utilization. The coefficients δ and γ determine this effect, indicating that energy consumption is lower when the warehouse is fully utilized.

The final component, ρ, represents the ratio between the coefficients of performance (COP) for refrigeration systems. It is given by the following equation:

$$\rho = \frac{COP_{Tr}}{COP_{Tw}} = \frac{T_r}{T_{hot} - T_r}\frac{T_{hot} - T_w}{T_w} \tag{38}$$

The maximum inventory level is defined in equation (23) and (24), and the inventory levels at time t, as mentioned in equations (14), (15), (22), and (25), are recorded as follows

$$I(t) = \begin{cases} \dfrac{P-D}{\alpha}[1-\exp(-\alpha t)], & if\ 0 \le t \le t_1 \\ \dfrac{D}{\alpha}\left[\dfrac{\exp(\alpha t_2) - \exp(\alpha t)}{\exp(\alpha t)}\right], & if\ 0 \le t \le t_2 \\ 0, \quad if\ t_1 + t_2 < t_3, t_4 \le t_c \end{cases} \tag{39}$$

The elements of (29) can be separated and developed as follows

$$\begin{aligned} EC_{ware.} &= \frac{E}{t_c}\left[\int_0^{t_c} \lambda I_{max}^{-\mu+1} \rho dt + \int_0^{t_c} \delta\left(\left(1 - \frac{I(t)}{I_{max}}\right)^{\gamma} \rho I_{max} dt\right]\right. \\ &= \frac{E}{t_c}[A(t) + \delta\rho I_{max} B(t)] \end{aligned} \tag{40}$$

Where

$$A(t) = \int_0^{t_c} \lambda I_{max}^{-\mu+1} \rho dt = \lambda\rho I_{max}^{-\mu+1} t|_0^{t_c} = \lambda\rho t_c\left[\frac{D}{\alpha}\left(\alpha t_2 + \frac{(\alpha t_2)^2}{2}\right)\right]^{-\mu+1}$$

As previously discussed, we once again adopt the simplified approximation method in order to reduce the complexity of the mathematical formulation and facilitate the solution process for the system of equations involved.

$$\begin{aligned} A(t) &= \lambda\rho t_c\left[Dt_2\left(1 + \frac{\alpha t_2}{2}\right)\right]^{-\mu+1} \\ &= \lambda\rho t_c\left[D\left(\frac{P-D}{P}\right) f t_c\right]^{-\mu+1} \end{aligned} \tag{41}$$

$$B(t) = B_1(t) + B_2(t) + B_{3,4}(t), \tag{42}$$

Where the expressions are computed as follows:

$$
\begin{aligned}
B_1(t) &= \int_0^{t_1} \left[1 - \frac{\frac{P-D}{\alpha}[1-\exp(-\alpha t)]}{I_{max}}\right]^{\gamma} dt \\
&= \int_0^{t_1} \left[1 - \frac{\frac{P-D}{\alpha}\left(\alpha t - \frac{(\alpha t)^2}{2}\right)}{I_{max}}\right]^{\gamma} dt \\
&= \int_0^{t_1} \left[1 - \frac{(P-D)t}{I_{max}}\right]^{\gamma} dt \\
&= -\frac{I_{max}}{(P-D)(\gamma+1)}\left[1 - \frac{(P-D)t}{I_{max}}\right]^{\gamma+1}\Bigg|_0^{t_1} \\
&= \frac{ft_c}{(\gamma+1)}\frac{D}{P} \\
B_2(t) &= \int_0^{t_2} \left[1 - \frac{\frac{D}{\alpha}\left[\frac{\exp(\alpha t_2)-\exp(\alpha t)}{\exp(\alpha t)}\right]}{I_{max}}\right]^{\gamma} dt \\
&= \int_0^{t_2} \left[1 - \frac{\frac{D}{\alpha}\left[-\alpha(t-t_2) + \frac{\alpha^2(t-t_2)^2}{2}\right]}{I_{max}}\right]^{\gamma} dt \\
&= \int_0^{t_2} \left[1 - \frac{D(t_2-t)}{I_{max}}\right]^{\gamma} dt \\
&= \frac{I_{max}}{D(\gamma+1)}\left[1 - \frac{Dt_2}{I_{max}} + \frac{Dt}{I_{max}}\right]^{\gamma+1}\Bigg|_0^{t_2} \\
&= \frac{ft_c}{(\gamma+1)}\left(\frac{P-D}{P}\right) \\
B_{3,4}(t) &= \int_{t_1+t_2}^{t_c} \left[1 - \frac{0}{I_{max}}\right]^{\gamma} dt = t_3 + t_4 \\
&= \frac{D}{P}(1-f)t_c + \frac{P-D}{P}(1-f)t_c \\
&= (1-f)t_c
\end{aligned}
$$

By substituting the results derived in equations (41) and (42) into equation (40), the average related-warehousing energy consumption cost can be reformulated in the following manner:

$$EC_{ware.} = \lambda\rho E\left[D\left(\frac{P-D}{P}\right)ft_c\right]^{-\mu+1} + \delta\rho t_c ED\left(\frac{P-D}{P}\right)\left(1-\frac{\gamma f}{\gamma+1}\right) \quad (43)$$

3.4 Objective function and constraints

The average total cost in this case is calculated as the sum of the Setup cost, Holding cost, Deterioration cost, Backorder cost, Average related-production energy consumption cost, Average related-warehousing energy consumption cost, as expressed in equation (42).

$$\begin{aligned} ATC\,(t_c, f, P) \; &= \frac{S}{t_c} + \frac{HDf^2t_c(P-D)\rho}{2P}\left(1+ft_c\alpha\frac{(P-2D)}{3P}\right) \\ &+ \frac{\alpha f^2 t_c CD(P-D)}{2P}\left(1+ft_c\alpha\frac{(P-2D)}{3P}\right) \\ &+ \frac{B(1-f)^2t_c}{2}D\left(1-\frac{D}{P}\right) \\ &+ \left(\frac{W}{P}+K\right)DE + WE\left(1-\frac{D}{P}\right) \\ &+ \lambda\rho E\left[D\left(\frac{P-D}{P}\right)ft_c\right]^{-\mu+1} \\ &+ \delta\rho t_c ED\left(\frac{P-D}{P}\right)\left(1-\frac{\gamma f}{\gamma+1}\right) \quad (44) \end{aligned}$$

Subject to the operational constraints:

$$P_{min} < P < P_{max}, \quad 0 < f < 1$$

The variations in internal temperature impact holding cost, embedding ρ in the total cost function. Furthermore, the inclusion of a production rate range $[P_{min}; P_{max}]$ serves two key purposes. First, it ensures the feasibility of the model, satisfying the condition $P > D$ as previously assumed. Second, it reflects a practical constraint commonly observed in real-world manufacturing, where production capacity is limited to a fixed operational range. By incorporating this bounded interval, the model remains both analytically tractable and aligned with actual industrial settings.

3.5 Resolution approach

The objective of the proposed EPQ model is to minimize the average total cost (ATC) function, as expressed in equation (36), by jointly determining the optimal values for three key decision variables:

- The production rate P
- The cycle time t_c
- The fraction of the cycle length with positive inventory f, where $f \in (0,1)$.

The optimization problem is thus formulated as follows:

$$Minimize\ ATC(t_c, P, f\ )$$

s.t.

$$P_{min} \leq P \leq P_{max}$$
$$0 < f < 1$$

We can use commercial solvers for NLP problems (e.g., LINGO, MATLAB) to find the two optimal values: $ATC(t_c, P_{min}, f)$ and $ATC\ (t_c, P_{max}, f)$. The optimal solution is the smallest value among these two values. Finally, we have the optimal decision values for t_c^*, P^*, f^*.

In this study, the DE algorithm is implemented using the `differential_evolution` function from the SciPy optimization module in Python. The DE process iteratively evolves a population of candidate solutions by applying mutation, crossover, and selection operations, converging towards a global minimum without requiring gradient information. This method is particularly suitable for the current model due to:

- The nonlinear interdependence of cost components,
- The presence of exponential decay,
- Multiple local minima resulting from trade-offs between energy cost and deterioration.

Figure 3.2 illustrates the overall procedure of the Differential Evolution (DE) algorithm used to solve the proposed nonlinear optimization problem. The flowchart reflects the population-based nature of DE and its iterative process for exploring the feasible solution space defined by the decision variables t_c, f and P.

The algorithm begins by defining system parameters such as setup cost (S), holding cost (H), demand rate (D), deterioration rate (α),...which are used to construct the cost function. Intermediate parameters $(e.g., \rho, \lambda)$ are then computed to capture warehouse-specific energy characteristics. A population of candidate solutions $X_i = (t_c, f, P)$ is initialized within the given bounds. For each individual in the population, the average total cost function $ATC(X_i)$ is evaluated. If a candidate violates the feasibility constraints $(e.g., f \notin (0,1),\ P < D)$, it is penalized by assigning a high cost (typically $ATC = \infty$).

The core mechanism of DE involves generating mutant vectors by combining other individuals in the population and performing crossover operations to produce trial solutions. These trial solutions are compared against their parents, and replacement occurs only if they yield a lower cost.

At each generation, the global best solution is updated if improvement is found. The process repeats until a stopping condition is met, typically when a maximum number of iterations is reached or when convergence is detected (e.g., no significant improvement in ATC).

This evolutionary strategy ensures a robust global search, avoids premature convergence, and is well-suited for the highly nonlinear cost landscape of the proposed EPQ model with energy and deterioration considerations.

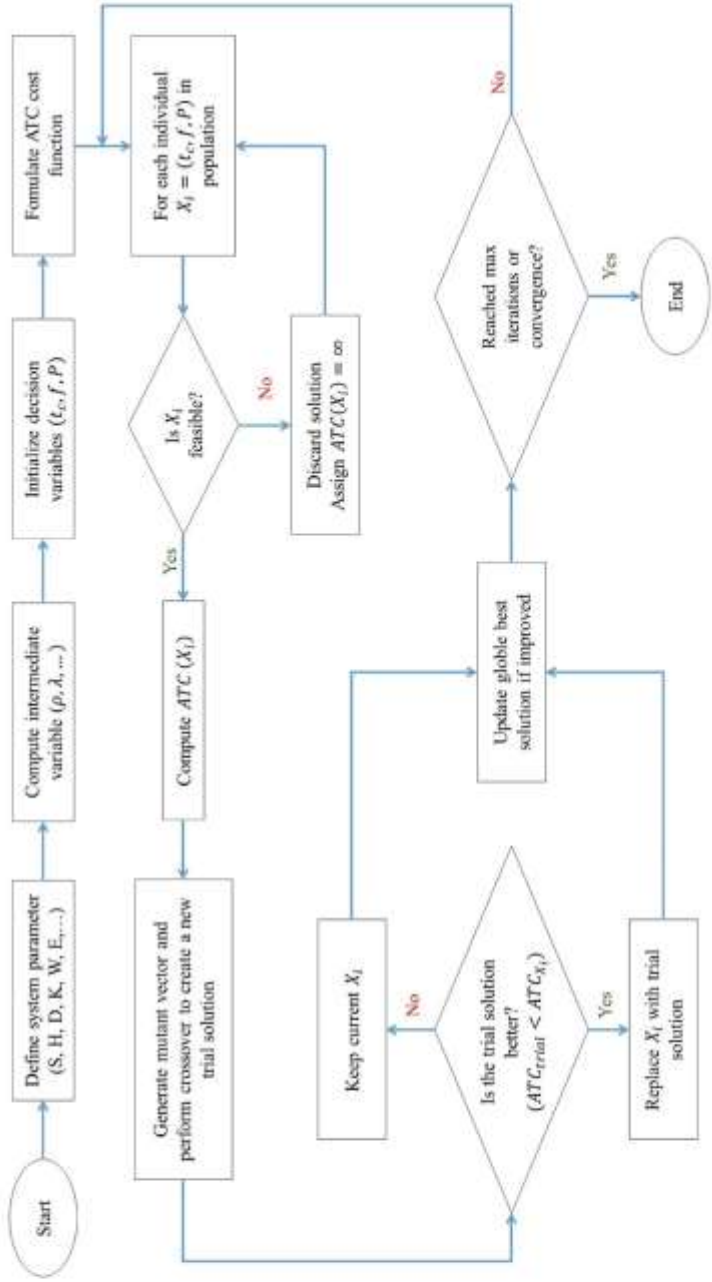

Figure 3.2 Optimization framework using Differential Evolution

3.6 Numerical analysis

A numerical analysis has been carried out to demonstrate the model's properties. The process includes:

- Implementing the resolution procedure for a specific case using existing research data.
- Analyzing the impact of energy components on the production inventory model by comparing the energy-based EPQ model with deterioration items to the traditional EPQ model with deterioration items.
- Conducting a sensitivity analysis to examine how input parameters influence the total cost and decision variables.

3.6.1 Numerical example

The input parameters associated with the manufacturing process, including demand rate (D), setup cost (S), holding cost (H), production rates range (P_{min}, P_{max}), idle energy consumption (W), energy requirement per unit produced (K), and electricity cost (E) were adopted from the study by Zanoni et al. [10] and the input parameters related to warehousing ($i.e. \lambda, \mu, \delta, \gamma$) have been obtained from Marchi et al. [38]. Additionally, the deterioration rate (α) and unit production cost (C) were sourced from Misra [25].

Table 3.1 summarizes the data set used in all the examples presented in this section.

Table 3.1 Input parameters for the numerical examples

D	100	$unit/h$	α	0.02	
S	300	$\$/h$	λ	$445 \cdot 10^{-4}$	
H	$1 \cdot 10^{-5}$	$\$/(unit \cdot h)$	μ	0.23	
W	100	kW	δ	$171.2 \cdot 10^{-4}$	
K	0.05	$kWh/unit$	γ	0.5	
E	0.2	$\$/(kWh)$	T_w	-18	$°C$
B	0.01	$\$/(unit \cdot h)$	T_r	6	$°C$
C	3	$\$/(unit \cdot h)$	T_{hot}	16	$°C$
P	$[200: 500]$	$unit/h$			

Using these parameters and the Differential Evolution algorithm introduced in **Section 3.5**, the optimal solution can be determined effectively. The optimal solution with the defined input parameter is illustrated in Figure 3.3.

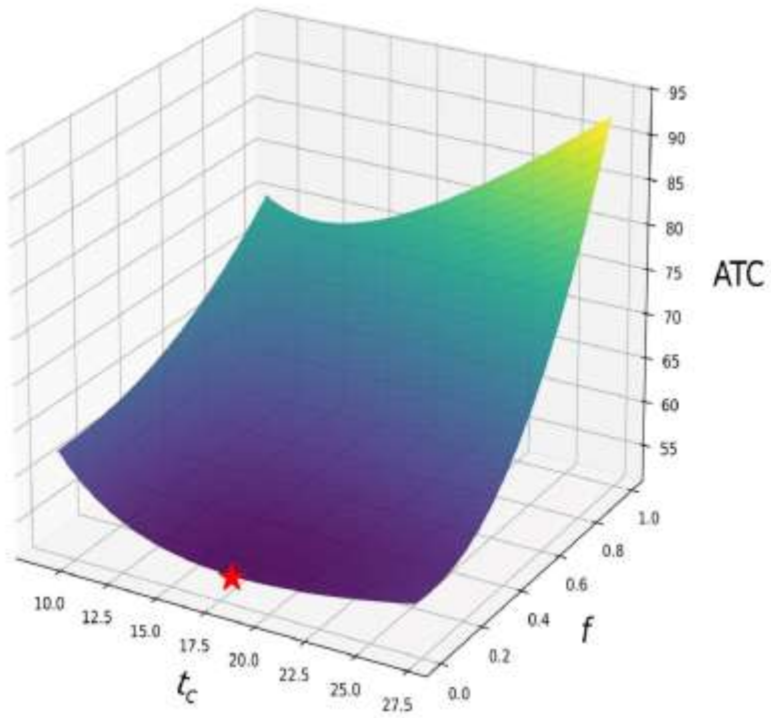

Figure 3.3 Total cost and optimal solution of fraction of period length and cycle time

Fig. 3.3 illustrates the concavity of the average total cost function with respect to P. Based on the solution approach presented in the previous section, the function $ATC(t_c, P, f)$ attains its minimum at $t_c^* = 18.451\ h,\ P^* = P_{min}, f^* = 0.057$ with the optimal value $ATC^*(t_c, P, f) = 53.678\ (\$/h)$. The optimal solution obtained will be used to compare with the traditional EPQ model considering deterioration, in order to evaluate its effectiveness.

3.6.2 Impact of energy – related factors on the traditional EPQ model with deterioration

To investigate the impact of energy factors on production and inventory storage, we analyze two models:

(1) The traditional EPQ model considering item deterioration **(EPQ-D)**

(2) An extended EPQ model incorporating energy considerations in both production and storage **(EEPQ-D)**.

A comparative analysis of these models is conducted to assess inventory policies and economic efficiency, providing a foundation for further discussion.

In the **EPQ-D** model, the average total cost function is denoted as $ATC_{NE}(t_c, P, f)$ with cost components defined by equations (31), (32), (33) and (34). Specifically, the objective function is expressed as:

$$ATC_{NE}(t_c,P,f) = \frac{S}{t_c} + \frac{HDf^2t_c(P-D)}{2P}\left(1+ft_c\alpha\frac{(P-2D)}{3P}\right) + \frac{\alpha f^2t_cCD(P-D)}{2P}\left(1+ft_c\alpha\frac{(P-2D)}{3P}\right) + \frac{B(1-f)^2t_c}{2}D\left(1-\frac{D}{P}\right) \quad (45)$$

The comparison results are presented in Table 3.2, with all input parameters specified in Table 2. The sum of setup cost and holding cost is labeled as Traditional cost ($C_{trad.}$), whereas the total energy-related costs in production and storage are denoted as Energy cost (EC) and Deterioration cost (DC).

Table 3.2 Optimal solutions and the average total cost components of two model comparison

	P^*	f^*	t_c^*	ATC	$C_{trad.}$	EC	DC
EEPQ-D	200	0.057	18.451	53.678	16.260	33.224	0.089
EPQ-D	200	0.143	37.412	62.174	8.026	46.136	1.142

Regarding result in Table 3.2, although both models achieve an optimal production quantity of $P^* = 200$, the **EEPQ-D** model exhibits a shorter production cycle (t_c) and a lower production time ratio (f) compared to the **EPQ-D** model. This means that while the number of setups increases, the continuous operating time of the machine is reduced, leading to significant energy savings. Notably, the deterioration cost in the **EEPQ-D** model is only 0.089 ($\$/h$) − 92.24% lower than the 1.142 ($\$/h$) observed in the **EPQ-D** model, since optimal solution of f is small, the deterioration time is also short, leading to a reduction in deterioration costs.

Table 3.3 Cost Efficiency of the Energy-Integrated Model for Deteriorating Products

	ATC	EC	DC
EEPQ - D	↓ 13.66%	↓ 27.98%	↓ 92.24%

Table 3.3 indicating the incorporating energy factors not only optimizes overall cost but also minimizes product deterioration, thereby preserving inventory value. Furthermore, although the traditional cost in the **EEPQ-D** model is 16.260 ($\$/h$), which is higher than the 8.026($\$/h$) in the **EPQ-D** model, the energy cost in the **EEPQ-D** model is substantially lower at 33.224 ($\$/h$) compared to 46.136 ($\$/h$) in the **EPQ-D** model (a reduction of 27.98%). This ontributes to a lower overall average total cost (ATC) of 53.678 ($\$/h$) for **EEPQ-D**, which is 13.66% less than the 62.174 ($\$/h$) for **EPQ-D**.

These findings demonstrate that integrating energy considerations into production and inventory decisions for deteriorating products offers clear economic benefits and improves energy efficiency, especially in environments where energy costs are increasingly significant.

3.6.3 Sensitivity analysis

In decision-making processes, uncertainties can lead to change in parameter values, affecting overall system performance. To evaluate the impact of such changes, sensitivity analysis is a useful approach that allows for informed decision-making. Using numerical examples from the previous section, this study examines the sensitivity of various parameters by modifying one at a time while keeping the others constant. The parameters are categorized into three distinct groups:

Group 1: General production parameters (S, H, C, B, D)

Group 2: Energy consumption and deterioration factors (K, W, E, α)

Group 3: Inventory storage energy parameters $(T_w, \rho, \mu, \lambda, \delta, \gamma)$

To ensure the model's relevance to dynamic business environments, sensitivity analysis is performed by varying input parameters within a range of –25% to +25%, using an increment of 5%. This range and step size were selected to realistically reflect the typical fluctuations observed in the market. For instance, electricity prices may rise by up to 5% within a short time frame, and such increases often accumulate over time. Therefore, using a 5% step not only offers analytical precision but also aligns with real-world economic variability.

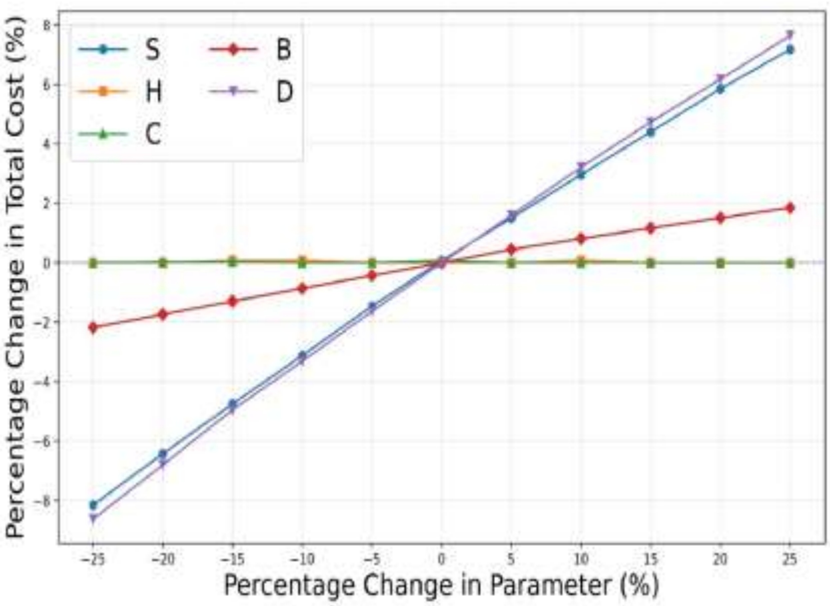

Figure 3.4 Impact of Group 1 Parameter Changes on Total Cost in Model 1

Sensitivity analysis shows that in Group 1, demand (D) and setup cost (S) has the most significant impact, with total cost increasing more than 6% when S rises by 25% and decreasing by 8% when S decreases. Backorder cost (B) also has a strong impact, while holding cost (H) and unit production cost (C) have little effect (<1%) and when B increases by 25%, increases total cost by about 2%. An observation of the sensitivity analysis table reveals that backorder cost (BC) has a noticeable impact on deterioration cost (DC), especially when it exceeds the threshold of 0.01. At this point, deterioration cost increases sharply, indicating a shift in the inventory policy that leads to longer holding times and greater spoilage.

We can see the result specifically in Table 3.4.

Table 3.4 Results of Sensitivity Analysis for Model 1 – Group 1 Parameter Changes

Parameter	Value	P^*	t_c^*	f^*	ATC	EC	DC
	225	200	15.923	0.001	49.268	31.164	0.00002
	240	200	16.481	0.043	50.243	31.861	0.04540
	255	200	16.951	0.001	51.093	31.821	0.00003
	270	200	17.495	0.052	52.009	32.572	0.06997
	285	200	17.921	0.001	52.814	32.439	0.00003
S	315	200	18.840	0.001	54.446	33.026	0.00003
	330	200	19.284	0.001	55.233	33.309	0.00003
	345	200	19.799	0.062	56.031	34.135	0.11343
	360	200	20.142	0.001	56.755	33.856	0.00003
	375	200	20.649	0.065	57.515	34.707	0.12898
	0.000075	200	18.386	0.001	53.640	32.736	0.00003
	0.00008	200	18.386	0.001	53.640	32.736	0.00003
	0.000085	200	18.451	0.057	53.678	33.225	0.08887
	0.00009	200	18.451	0.057	53.678	33.225	0.08876
	0.000095	200	18.386	0.001	53.640	32.736	0.00003
H	0.000105	200	18.386	0.001	53.640	32.736	0.00003
	0.00011	200	18.451	0.056	53.678	33.224	0.08829
	0.000115	200	18.386	0.001	53.640	32.736	0.00003
	0.00012	200	18.386	0.001	53.640	32.736	0.00003
	0.000125	200	18.386	0.001	53.640	32.736	0.00003
	2.25	200	18.386	0.001	53.640	32.736	0.00002
C	2.4	200	18.498	0.083	53.651	33.392	0.15378
	2.55	200	18.485	0.076	53.660	33.347	0.13549

	2.7	200	18.386	0.001	53.640	32.736	0.00003
	2.85	200	18.386	0.001	53.640	32.736	0.00003
	3.15	200	18.442	0.051	53.682	33.184	0.07436
	3.3	200	18.386	0.001	53.640	32.736	0.00003
	3.45	200	18.386	0.001	53.640	32.736	0.00003
	3.6	200	18.386	0.001	53.640	32.736	0.00003
	3.75	200	18.386	0.001	53.640	32.736	0.00003
	0.0075	200	19.069	0.001	52.472	33.172	0.00003
	0.008	200	18.926	0.001	52.709	33.081	0.00003
	0.0085	200	18.787	0.001	52.944	32.992	0.00003
	0.009	200	18.650	0.001	53.178	32.905	0.00003
	0.0095	200	18.517	0.001	53.410	32.820	0.00003
B	0.0105	200	18.258	0.001	53.869	32.655	0.00003
	0.011	200	18.255	0.081	54.076	33.220	0.17863
	0.0115	200	18.160	0.090	54.266	33.202	0.22249
	0.012	200	18.068	0.099	54.451	33.181	0.26686
	0.0125	200	17.977	0.108	54.633	33.157	0.31193
	75	200	21.230	0.001	49.018	30.914	0.00002
	80	200	20.601	0.043	50.043	31.661	0.04541
	85	200	19.943	0.001	50.943	31.671	0.00003
	90	200	19.439	0.052	51.909	32.472	0.06996
	95	200	18.926	0.054	52.805	32.853	0.07956
D	105	210	17.943	0.001	54.496	33.076	0.00003
	110	220	17.531	0.001	55.333	33.409	0.00003
	115	230	17.146	0.001	56.152	33.735	0.00003
	120	240	16.785	0.001	56.955	34.056	0.00003
	125	250	16.446	0.001	57.742	34.371	0.00003

In Group 2, energy cost (E) has the strongest impact, increasing by +15%, confirming the significance of energy consumption in the model. When combined with the influence of setup cost, as analyzed earlier, this finding reinforces the practical relevance of the model. It highlights that selecting machines with lower energy consumption is a key strategy for optimizing overall costs. When energy cost (E) are low, firms tend to adopt longer production cycles to take advantage of economies of scale. However, this strategy results in prolonged inventory holding, thereby increasing the risk of product deterioration. A recommended approach is to implement production cycle constraints or adopt a "Just-in-Time" model in low-energy-cost environments to mitigate spoilage risks. Following energy cost, idle energy consumption (W) also plays a crucial role, emphasizing the importance of effectively managing standby energy usage. Meanwhile, the impact of α (the deterioration rate) is negligible, as the model optimizes solutions to minimize production and consumption time.

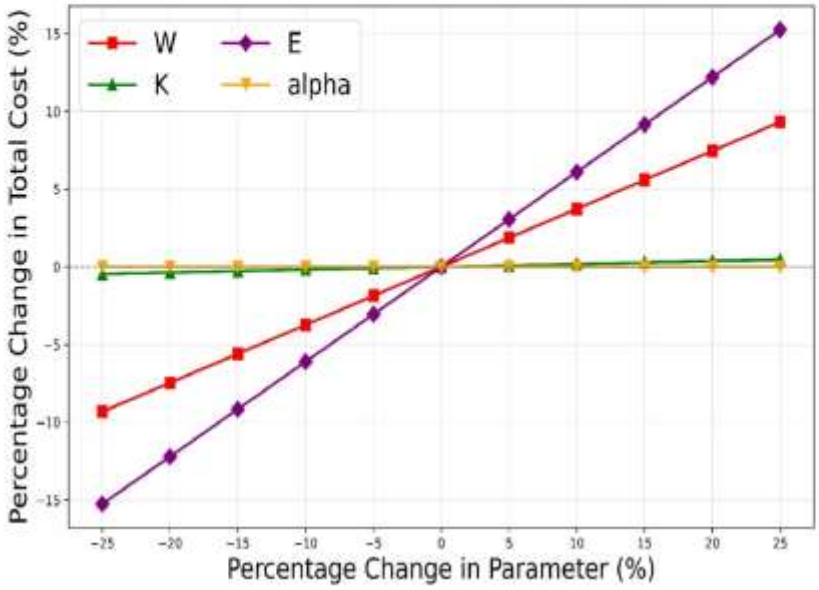

Figure 3.5 Impact of Group 2 Parameter Changes on Total Cost in Model 1

To better illustrate the impact of parameter changes in Group 2, the results of the sensitivity analysis are presented in Table 3.5 below.

Table 3.5 Results of Sensitivity Analysis for Model 1 – Group 2 Parameter Changes

Parameter	Value	P^*	t_c^*	f^*	ATC	EC	DC
	75	200	18.451	0.057	48.678	28.224	0.08852
	80	200	18.386	0.001	49.640	28.736	0.00003
	85	200	18.440	0.057	50.678	30.217	0.08832
	90	200	18.386	0.001	51.640	30.736	0.00003
	95	200	18.386	0.001	52.640	31.736	0.00003
W	105	200	18.386	0.001	54.640	33.736	0.00003
	110	200	18.451	0.057	55.678	35.224	0.08853
	115	200	18.451	0.057	56.678	36.224	0.08853
	120	200	18.386	0.001	57.640	36.736	0.00003
	125	200	18.386	0.001	58.640	37.736	0.00003
	0.0375	200	18.451	0.057	53.428	32.974	0.08853

	0.04	200	18.451	0.057	53.478	33.024	0.08853
	0.0425	200	18.386	0.001	53.490	32.586	0.00003
	0.045	200	18.451	0.057	53.578	33.124	0.08853
	0.0475	200	18.386	0.001	53.590	32.686	0.00003
K	0.0525	200	18.451	0.057	53.728	33.274	0.08852
	0.055	200	18.386	0.001	53.740	32.836	0.00003
	0.0575	200	18.386	0.001	53.790	32.886	0.00003
	0.06	200	18.386	0.001	53.840	32.936	0.00003
	0.0625	200	18.386	0.001	53.890	32.986	0.00003
	0.15	200	20.516	0.092	45.185	26.071	0.25960
	0.16	200	20.050	0.087	46.915	27.543	0.22527
	0.17	200	19.614	0.081	48.628	28.996	0.19173
	0.18	200	19.204	0.074	50.327	30.430	0.15852
	0.19	200	18.726	0.001	51.998	31.305	0.00003
E	0.21	200	18.065	0.001	55.272	34.157	0.00003
	0.22	200	17.759	0.001	56.893	35.570	0.00003
	0.23	200	17.468	0.001	58.505	36.973	0.00003
	0.24	200	17.192	0.001	60.108	38.369	0.00003
	0.25	200	16.928	0.001	61.703	39.757	0.00003
	0.015	200	18.513	0.091	53.641	33.440	0.17389
	0.016	200	18.498	0.083	53.651	33.392	0.15378
	0.017	200	18.485	0.076	53.660	33.347	0.13549
	0.018	200	18.472	0.069	53.667	33.305	0.11872
	0.019	200	18.386	0.001	53.640	32.736	0.00003
α	0.021	200	18.386	0.001	53.640	32.736	0.00003
	0.022	200	18.386	0.001	53.640	32.736	0.00003
	0.023	200	18.386	0.001	53.640	32.736	0.00003
	0.024	200	18.386	0.001	53.640	32.736	0.00003
	0.025	200	18.386	0.001	53.640	32.736	0.00003

In Group 3, sensitivity analysis reveals that energy efficiency (ρ), warehouse temperature (T_w), and storage utilization factors (δ, γ) exhibit a significant impact on the total cost, with variations ranging from 4% to 6%. In contrast, the effects of μ and λ are negligible. These findings highlight that ambient control, particularly warehouse temperature – is a critical factor influencing cost performance. Moreover, the inventory level, which directly affects both storage energy consumption and deterioration, emerges as a key driver in maintaining overall cost efficiency. Therefore, optimizing warehouse conditions and managing inventory levels effectively are essential for minimizing total operational costs in sustainable inventory systems.

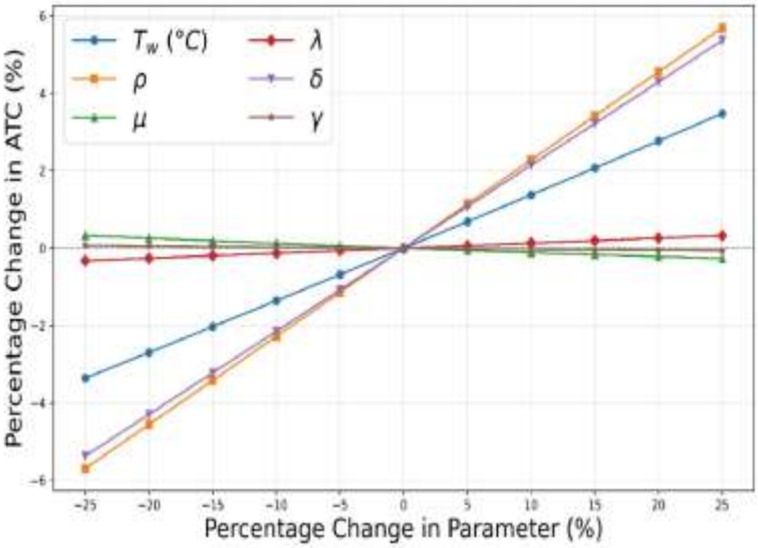

Figure 3.6 Impact of Group 3 Parameter Changes on Total Cost in Model 1

The significant influence of the parameters in Group 3 is specifically illustrated in the results presented below Table 3.6.

Table 3.6 Results of Sensitivity Analysis for Model 1 – Group 3 Parameter Changes

Parameter	Value	P^*	t_c^*	f^*	ATC	EC	DC
T_w	-13.5	200	19.592	0.080	51.813	32.167	0.19043
	-14.4	200	19.351	0.077	52.191	32.392	0.17102
	-15.3	200	19.118	0.073	52.566	32.612	0.15146
	-16.2	200	18.890	0.068	52.940	32.825	0.13155
	-17.1	200	18.587	0.001	53.287	32.510	0.00003
	-18.9	200	18.191	0.001	53.991	32.961	0.00003
	-19.8	200	18.000	0.001	54.342	33.184	0.00003
	-20.7	200	17.813	0.001	54.691	33.405	0.00003
	-21.6	200	17.631	0.001	55.038	33.624	0.00003
	-22.5	200	17.453	0.001	55.385	33.842	0.00003
ρ	2.790	200	20.516	0.092	50.435	31.322	0.26052
	2.976	200	20.050	0.087	51.115	31.744	0.22593
	3.162	200	19.614	0.081	51.778	32.147	0.19218
	3.348	200	19.204	0.074	52.426	32.531	0.15879

	3.534	200	18.726	0.001	53.048	32.355	0.00003
	3.906	200	18.065	0.001	54.222	33.107	0.00003
	4.092	200	17.759	0.001	54.793	33.470	0.00003
	4.278	200	17.469	0.001	55.355	33.823	0.00003
	4.464	200	17.192	0.001	55.908	34.169	0.00003
	4.650	200	16.928	0.001	56.453	34.507	0.00003
μ	0.173	200	18.385	0.001	53.640	32.735	0.00003
	0.184	200	18.386	0.001	53.640	32.736	0.00003
	0.196	200	18.386	0.001	53.640	32.736	0.00003
	0.207	200	18.386	0.001	53.640	32.736	0.00003
	0.219	200	18.386	0.001	53.640	32.736	0.00003
	0.242	200	18.387	0.001	53.640	32.736	0.00003
	0.253	200	18.531	0.087	53.600	33.336	0.20808
	0.265	200	18.554	0.097	53.555	33.341	0.26324
	0.276	200	18.604	0.107	53.507	33.350	0.31754
	0.288	200	18.639	0.115	53.457	33.340	0.36979
λ	0.033	200	18.600	0.108	53.439	33.283	0.32772
	0.036	200	18.579	0.100	53.495	33.306	0.28041
	0.038	200	18.547	0.092	53.547	33.311	0.23334
	0.040	200	18.516	0.082	53.596	33.305	0.18671
	0.042	200	18.485	0.071	53.640	33.281	0.13956
	0.047	200	18.386	0.001	53.642	32.737	0.00003
	0.049	200	18.385	0.001	53.643	32.738	0.00003
	0.051	200	18.384	0.001	53.645	32.740	0.00003
	0.053	200	18.384	0.001	53.646	32.741	0.00003
	0.056	200	18.383	0.001	53.648	32.742	0.00003
δ	0.013	200	20.297	0.001	50.569	30.724	0.00003
	0.014	200	19.867	0.001	51.208	31.151	0.00003
	0.015	200	19.463	0.001	51.834	31.565	0.00003
	0.015	200	19.084	0.001	52.448	31.966	0.00003
	0.016	200	18.780	0.050	53.096	32.813	0.07002
	0.018	200	18.138	0.062	54.249	33.615	0.10502
	0.019	200	17.760	0.001	54.790	33.467	0.00003
	0.020	200	17.470	0.001	55.351	33.820	0.00003
	0.021	200	17.194	0.001	55.903	34.165	0.00003

	0.021	200	17.028	0.081	56.433	35.050	0.16558
	0.375	200	18.386	0.001	53.641	32.737	0.00003
	0.400	200	18.386	0.001	53.641	32.737	0.00003
	0.425	200	18.386	0.001	53.640	32.736	0.00003
	0.450	200	18.386	0.001	53.640	32.736	0.00003
	0.475	200	18.440	0.051	53.686	33.192	0.07176
γ	0.525	200	18.386	0.001	53.640	32.736	0.00003
	0.550	200	18.470	0.065	53.663	33.267	0.11761
	0.575	200	18.479	0.069	53.655	33.282	0.13106
	0.600	200	18.488	0.072	53.647	33.295	0.14405
	0.625	200	18.505	0.075	53.638	33.312	0.15683

In conclusion, the sensitivity analysis confirms that both economic and environmental factors significantly influence the total system cost. Among them, demand, setup cost, energy cost, and warehouse temperature stand out as the most impactful parameters. These results emphasize the importance of integrated decision-making that accounts not only for traditional cost components but also for energy efficiency and environmental control. Optimizing these critical parameters is essential for enhancing cost-effectiveness and achieving long-term sustainability in production-inventory systems.

3.7 Conclusion

Chapter 3 successfully developed an extended Economic Production Quantity (EPQ) model that integrates constant deterioration and energy consumption, addressing the real-world challenges of managing perishable inventory in energy-intensive systems. The model incorporates five major cost components: setup cost, holding cost, deterioration cost, backorder cost, and energy costs across three operational stages—active production, machine idle time, and warehousing. By modeling deterioration using a Weibull distribution with $\beta = 1$ (exponential decay), the approach retains analytical tractability while realistically representing quality degradation over time.

The optimization problem was solved using the Differential Evolution (DE) algorithm implemented in Python, yielding a globally optimal solution with a production cycle of approximately 18 hours, a minimal positive inventory fraction ($f = 0.057$), and a production rate at the lower bound of the feasible domain. Comparative analysis revealed that integrating energy considerations reduced the average total cost by over 13%, energy cost by nearly 30%, and deterioration cost by over 90% compared to the traditional EPQ-D model. Sensitivity analysis identified market demand, setup cost, electricity price, and warehouse temperature as the most influential parameters.

These results provide managerial insights for sustainable inventory control: firms should invest in accurate demand forecasting, energy-efficient equipment, and optimized production scheduling, while carefully managing temperature settings in cold storage to achieve both economic and environmental objectives.

CHAPTER 4 EXTENDED EPQ MODEL INCORPORATING VARIABLE DETERIORATION RATES AND ENERGY EFFICIENCY CONSIDERATIONS

In **Chapter 3**, an Economic Production Quantity (EPQ) model incorporating energy consumption and a constant deterioration rate was developed to reflect cost behavior in practical production settings. Building upon that foundation, **Chapter 4** extends the model by introducing a more advanced formulation in which the deterioration rate varies over time, modeled using the Weibull distribution. This enhancement allows the model to more accurately capture the dynamic quality degradation observed in perishable products. Following the model formulation, numerical simulations and sensitivity analyses are conducted to evaluate the model's performance under various operating conditions. These analyses aim to demonstrate the practical value and applicability of the proposed model in real-world production and storage systems.

4.1 Introduction and problem statement

In practice, several categories of goods such as food, pharmaceuticals, blood, and biological materials; do not degrade at a constant rate, but instead deteriorate over time in ways that are influenced by environmental conditions and product-specific properties. The EPQ model with a constant deterioration rate, as developed in **Chapter 3**, is only suitable for items whose quality declines at a stable pace over time. However, to support the global trend toward energy-efficient and quality-focused inventory planning, it is necessary to build a more generalized model.

In this extended model, the deterioration process is described using the Weibull distribution with a shape parameter $\beta \neq 1$, allowing for greater flexibility in modeling various types of degradation behavior, including accelerating or decelerating deterioration. The model retains the same energy considerations as in the previous chapter (see Figure 3.1), accounting for energy consumption across three operational phases:

- During active production, where energy usage depends on the production rate;
- During standby periods, when the machine is idle but still consumes base-load energy;
- During storage, where energy is required to maintain controlled environmental conditions (e.g., refrigeration).

The primary objective of the model is to minimize the total average system cost, which consists of:

- Setup cost per production cycle,
- Inventory holding cost,
- Deterioration cost with a variable deterioration rate,
- Shortage cost due to backordered items,
- Energy costs incurred during production, standby, and storage phases.

4.2 Notations and assumptions

4.2.1 Notations

The following notation is used to describe the model:

Parameters

D : Demand rate (unit/h)

S : Setup cost (\$/setup)

H : Holding cost per unit (\$/(unit.h))

C : Unit production cost (\$/(unit.h))

B : Backorder cost (\$/(unit.h))

W : Idle energy consumption (kW)

K : Energy consumption for one unit producing (kWh/unit)

E : Energy cost (\$/kWh)

$L(t)$: Deterioration rate, given by $\alpha\beta t^{\beta-1}$ where $\alpha, \beta, t > 0$. When $\beta < 1$, the rate of deterioration is decreasing with t and when $\beta > 1$, it is increasing with t;

T_w : Expected warehouse temperature (°C)

T_r : Referenced warehouse temperature (°C)

T_{hot} : Outside warehouse temperature (°C)

ρ : Coefficient linking SEC to various storage temperatures

λ. μ : Positive coefficients dependent on the characteristics of the warehouse, where $\mu \in (0, 1)$

δ, γ : Positive coefficients dependent on the filling level of the warehouse.

Dependent variables

t_1 : Time of production time (h)

t_2 : Time of non-production time (h)

t_3 : Time of consumption sub-time in shortage period (h)

t_4 : Time of production sub-time in shortage period (h)

I : Inventory level at time t (unit)

I_{max} : Maximum storage capacity of the warehouse (unit)

I_b : Stockout demand (units)

Decision variables

P : Production rate (unit/h)

t_c : Cycle time (h)

f : Fraction of period length with positive inventory level, $f \in (0, 1]$

4.2.2 Assumptions

The assumption of an inventory model for product life cycle are as follows:

- Demand is known and has a constant rate.
- The production rate is limited, greater than demand rate ($P > D$) and confine within a specific range [P_{min}, P_{max}].
- Changes to the production rate are implemented only at the beginning of the production cycle.
- Stortages are allowed with complete backlogging.
- Lead time is neglible.
- The units are available for satisfying demand immediately after their production.
- The deterioration process follows a variable-rate Weibull distribution, under the assumption that items begin to deteriorate immediately upon entering inventory.
- Throughout each cycle, no replacement or repair of deteriorated items is performed.
- The cost of a deteriorated unit is constant and equal to the unit production cost C. This will account for the salvage value, if any.
- The machine remains in an idle state during the non-production period.

4.3 Mathematical model development

The structural layout of this model remains the same as that shown in **Figure 3.1**, with the key distinction being the deterioration rate function. In this chapter, the deterioration process is represented by a time-dependent Weibull function defined as:

$$L(t) = \alpha\beta t^{\beta-1}$$

where α is the scale parameter and β is the shape parameter that determines the rate of deterioration over time.

Two distinct deterioration behaviors can be observed based on the value of β:

- When $\beta < 1$, the product deteriorates rapidly at the beginning, and then the deterioration rate gradually slows down over time. This pattern is characteristic of certain food products and biological materials.
- When $\beta > 1$, the deterioration process accelerates over time, indicating that the item becomes increasingly fragile as it ages.

Let $L(t)$ denote the deterioration rate at time ttt, and let the inventory levels at different stages of the production cycle t_1, t_2, t_3, t_4 be defined as:

$$\frac{dI_1}{dt} + L(t)I_1 = P - D, \quad 0 \le t \le t_1 \tag{46}$$

$$\frac{dI_2}{dt} + L(t)I_2 = -D, \quad 0 \le t \le t_2 \tag{47}$$

$$\frac{dI_3}{dt} = -D, \quad 0 \le t \le t_3 \tag{48}$$

$$\frac{dI_4}{dt} = P - D, \quad 0 \le t \le t_4 \tag{49}$$

The solutions to the above differential equations using Spiegel (1960) and the boundary conditions. So that, at t=0, I_1=0, the initial inventory, and at $t = t_1, I_2 = I_{max}$; at $t = t_2$, $I_3 = 0$ and at $t = t_3, I_4 = I_b$, are

$$I_1 = \frac{\int_0^t (P-D) epx(\int L(t)dt)dt}{\exp(\int_0^t \int L(t)dt)}, \quad 0 \le t \le t_1 \tag{50}$$

$$I_2 = \frac{I_{max} + \int_0^t (-D) epx(\int L(t)dt)dt}{\exp(\int_0^t L(t)dt)}, \quad 0 \le t \le t_2 \tag{51}$$

$$I_3 = -Dt, \quad 0 \le t \le t_3 \tag{52}$$

$$I_4 = (P-D)t, \quad 0 \le t \le t_4 \tag{53}$$

Now at $t = t_2, I_2 = 0$, hence

$$I_{max} = \int_0^{t_2} D\, exp\left(\int L(t)dt\right) dt \tag{54}$$

Substituting this in equa. (51) yields

$$I_2 = \frac{\int_0^{t_2} D\, exp(\int L(t)dt)dt + \int_0^t (-D) epx(\int L(t)dt)}{\exp\left(\int_0^t L(t)dt\right)} \tag{55}$$

With $L(t) = \alpha\beta t^{\beta-1}$, where α, β are some constants determined by the deterioration process. By subtituting this value of $L(t)$ in (51), (54), (55) we have

$$I_1 = \frac{\int_0^t (P - D) epx(\alpha t^\beta)}{\exp(\alpha t^\beta)} \quad (56)$$

$$I_{max} = \int_0^{t_2} D\, exp(\alpha t^\beta) dt \quad (57)$$

$$I_2 = \frac{\int_0^{t_2} D\, exp(\alpha t^\beta) dt + \int_0^t (-D) epx(\alpha t^\beta) dt}{\exp(\alpha t^\beta)} \quad (58)$$

By using the series approximation and neglecting second or higher order of α terms, we have,

$$I_1 = (P - D)\left(t + \frac{\alpha}{\beta + 1} t^{\beta+1}\right)(1 - \alpha t^\beta) \quad (59)$$

$$I_{max} = D\left(t_2 + \frac{\alpha}{\beta + 1} t_2^{\beta+1}\right) \quad (60)$$

$$I_2 = D(1 - \alpha t^\beta)\left[\left(t_2 + \frac{\alpha}{\beta + 1} t_2^{\beta+1}\right) - \left(t + \frac{\alpha}{\beta + 1} t^{\beta+1}\right)\right] \quad (61)$$

As previously discussed, while t_1 and t_2 are theoretically well-defined and mathematically satisfactory, their practical computation is highly challenging, as the integrals involved in their expressions do not admit closed-form solutions. This complexity makes direct evaluation infeasible in most real-world applications.

Therefore, to overcome this limitation, mathematical approximation techniques can be employed to linearize the relationships, following the approach previously applied in the constant deterioration case. By doing so, the time intervals t_1, t_2, t_3, t_4 can be expressed using tractable approximate formulas.

$$t_1 = \frac{D}{P} f t_c \quad (62)$$

$$t_2 = \frac{P - D}{P} f t_c \quad (63)$$

$$t_3 = \frac{P - D}{P} (1 - f) t_c \quad (64)$$

$$t_4 = \frac{D}{P} (1 - f) t_c \quad (65)$$

Next, these dependent variables are used to construct the component costs, including setup cost, holding cost, deterioration cost, backorder cost, and energy consumption during both the production and storage phases. These components are then combined to formulate the total average cost function of the system.

4.3.1 Setup cost

This cost component remains identical to the constant deterioration case presented in **Chapter 3**, as it represents the traditional cost structure. In this chapter, only the deterioration aspect of the inventory is modified and extended. Therefore, the same formulation as in equation (31) is used to describe this cost.

$$SC = \frac{S}{t_c}$$

4.3.2 Holding cost

In this extended model, the holding cost component differs from the constant deterioration case due to the introduction of a time-dependent deterioration rate. Specifically, items in stock are assumed to deteriorate continuously over time, with the deterioration process following a Weibull distribution.

The accumulation of holding cost occurs during the time intervals in which inventory is present, namely during t_1 and t_2. Let I_1 and I_2 denote the inventory levels over these respective phases. The holding cost per unit time is then computed by integrating the product of the deterioration rate and the corresponding inventory level across both intervals. Mathematically, the total holding cost is formulated as:

$$HC = \frac{H}{t_c}\left[\int_0^{t_1} I_1 dt + \int_0^{t_2} I_2 dt\right]$$

$$= \frac{H}{t_c}\begin{bmatrix} \int_0^{t_1}(P-D)\left(t+\frac{\alpha}{\beta+1}t^{\beta+1}\right)\left(1-\alpha t^{\beta}\right) \\ +\int_0^{t_2} D(1-\alpha t^{\beta})\left[\left(t_2+\frac{\alpha}{\beta+1}t_2^{\beta+1}\right)-\left(t+\frac{\alpha}{\beta+1}t^{\beta+1}\right)\right] \end{bmatrix}$$

$$= \frac{H}{t_c}\begin{bmatrix} (P-D)\left(\frac{t_1^2}{2}-\frac{\alpha\beta}{\beta+1}\frac{t_1^{\beta+2}}{\beta+2}-\frac{\alpha^2}{(\beta+1)^2}\frac{t_1^{2\beta+2}}{2}\right) \\ +D\left(\frac{t_2^2}{2}+\frac{\alpha\beta}{\beta+1}\frac{t_2^{\beta+2}}{\beta+2}-\frac{\alpha^2}{(\beta+1)^2}\frac{t_1^{2\beta+2}}{2}\right) \end{bmatrix}$$

$$= \frac{H}{t_c}\begin{bmatrix} \frac{f^2t_c^2}{2}\frac{(P-D)D}{P} \\ +\frac{\alpha\beta}{(\beta+1)(\beta+2)}\frac{(ft_c)^{\beta+2}}{P^{\beta+2}}\left(D(P-D)^{\beta+2}-(P-D)D^{\beta+2}\right) \\ -\frac{\alpha^2}{2(\beta+1)^2}\frac{(ft_c)^{2\beta+2}}{P^{2\beta+2}}\left(D^{2\beta+2}(P-D)+D(P-D)^{2\beta+2}\right) \end{bmatrix} \quad (66)$$

4.3.3 Deterioration cost

With the same structural formulation, the deterioration cost per unit time is determined by integrating the quantity of deteriorated items across the two inventory phases, then multiplying the result by the unit deterioration cost. This yields the following expression:

$$DC = \frac{C}{t_c}\left[\int_0^{t_1} L(t) I_1 dt + \int_0^{t_2} L(t) I_2 dt\right]$$

where C denotes the unit production cost, we will lost if the item is completely deteriorating, and I_1, I_2 represent the inventory levels over the time intervals t_1 and t_2 respectively. To better capture the time-varying nature of product deterioration, as discussed earlier in this chapter, the deterioration rate is defined by a Weibull distribution in the form $L(t) = \alpha\beta t^{\beta-1}$. Substituting this into the equation above allows the model to represent both decelerating deterioration (when $\beta < 1$) and accelerating deterioration (when $\beta > 1$), the deterioration cost becomes:

$$= \frac{C}{t_c}\left[\begin{array}{c} \int_0^{t_1} \alpha\beta t^{\beta-1}(P-D)\left(t + \frac{\alpha}{\beta+1}t^{\beta+1}\right)\left(1-\alpha t^{\beta}\right)dt \\ + \int_0^{t_2} \alpha\beta t^{\beta-1} D\left(1-\alpha t^{\beta}\right)\left[\left(t_2 + \frac{\alpha}{\beta+1}t_2^{\beta+1}\right) - \left(t + \frac{\alpha}{\beta+1}t^{\beta+1}\right)\right] \end{array}\right]$$

$$= \frac{C}{t_c}\left[\begin{array}{c} (P-D)\left(\frac{\alpha\beta t_1^{\beta+1}}{\beta+1} - \frac{\alpha^2\beta^2 t^{2\beta+1}}{(\beta+1)(2\beta+1)} - \frac{\alpha^3\beta t^{3\beta+1}}{(\beta+1)(3\beta+1)}\right) \\ +D\left(\frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{\alpha^2 t^{2\beta+1}}{2(2\beta+1)} - \frac{\alpha^3 t^{3\beta+1}}{(3\beta+1)}\right) \end{array}\right]$$

$$= \frac{C}{t_c}\left[\begin{array}{c} \frac{\alpha}{\beta+1}\frac{(ft_c)^{\beta+1}}{P^{\beta+1}}\left(D(P-D)^{\beta+1} + (P-D)\beta D^{\beta+1}\right) \\ + \frac{\alpha^2}{2\beta+1}\frac{(ft_c)^{2\beta+1}}{P^{2\beta+1}}\left(\frac{D}{2}(P-D)^{2\beta+1} - (P-D)D^{2\beta+1}\frac{\beta^2}{\beta+1}\right) \\ - \frac{\alpha^3}{(3\beta+1)}\frac{(ft_c)^{3\beta+1}}{P^{3\beta+1}}\left(D(P-D)^{3\beta+1} + D^{3\beta+1}(P-D)\frac{\beta}{(\beta+1)}\right) \end{array}\right] \quad (67)$$

4.3.4 Backorder cost

The backorder cost is calculated based on the time intervals during which demand is unmet, specifically t_3 and t_4, along with the corresponding levels of backordered inventory. However, since the current model only modifies the deterioration behavior of the product and does not alter the demand structure or the timing of shortages, the formulation of the backorder cost remains unchanged from the previous model.

As a result, the structure of this cost component is preserved, and it can be expressed in the same form as equation (34) presented earlier.

$$BC = \frac{B}{t_c}\left[\int_0^{t_3} I_3 dt + \int_0^{t_4} I_4 dt\right]$$
$$= \frac{B(1-f)^2 t_c}{2} D\left(1 - \frac{D}{P}\right),$$

4.3.5 Average related – production energy consumption

As discussed in Chapter 3, energy consumption in the production-inventory system arises from two primary sources:

- During the production phases t_1 and t_4, when the machine is operating to manufacture products;
- During the non-production phases t_2 and t_3, when the machine remains powered on but is not engaged in production (i.e., standby mode).

These two operational phases incur distinct types of energy costs, which are modeled using the Specific Energy Consumption (SEC) framework. The total average energy cost per unit time is calculated as the sum of energy usage across both production and non-production intervals.

Since the underlying definitions and relationships of the dependent variables t_1, t_2, t_3, t_4 remain unchanged in this extended model, the structure of the energy cost functions also remains the same. Specifically, the energy cost during production is computed using equation (35),

$$EC_{prod.} = \left(\frac{W}{P} + K\right) DE$$

and while the energy cost during non-production (idle) periods is given by equation (36), as presented in the previous chapter.

$$EC_{nonp.} = \frac{WE}{t_c}(t_2 + t_3)$$
$$= WE\left(1 - \frac{D}{P}\right)$$

4.3.6 Average related – warehousing energy consumption

A primary objective of this model is to incorporate the energy consumption required to preserve perishable products, which is closely linked to maintaining appropriate storage temperatures. In this context, the energy used in warehousing depends on two main factors:

- The target temperature level needed to preserve the quality of the stored items, and

- The inventory level at each point in time.

Perishable products typically require stricter environmental conditions, particularly temperature control, to prevent rapid deterioration. As such, ensuring an appropriate ambient temperature tailored to product characteristics, along with efficient inventory level management, plays a critical role in optimizing energy usage.

Given the previously established linear approximation between energy consumption and inventory level, this modeling approach remains valid and appropriate even when deterioration follows a Weibull distribution. Therefore, the energy cost associated with storage operations can be formulated using the same structure as in the constant deterioration model in equation (43).

$$EC_{ware.} = \lambda\rho E\left[D\left(\frac{P-D}{P}\right)ft_c\right]^{-\mu+1} + \delta\rho t_c ED\left(\frac{P-D}{P}\right)\left(1-\frac{\gamma f}{\gamma+1}\right)$$

4.4 Objective function

The average total cost in this case is computed as the sum of all relevant cost components, including: Setup cost, Holding cost, Deterioration cost, Backorder cost, Average energy consumption during production, and Average energy consumption during warehousing. These components are formulated respectively in equations (31), (34), (35), (36), (66), and (67), and are integrated into the total cost function presented in equation (68).

$$ATC\ (t_c, f, P) = \frac{S}{t_c} + \frac{B(1-f)^2 t_c}{2} D\left(1-\frac{D}{P}\right)$$

$$+\frac{H\rho}{t_c}\left[\begin{array}{c} \dfrac{f^2t_c^2}{2}\dfrac{(P-D)D}{P} \\ +\dfrac{\alpha\beta}{(\beta+1)(\beta+2)}\dfrac{(ft_c)^{\beta+2}}{P^{\beta+2}}\left(D(P-D)^{\beta+2}-(P-D)D^{\beta+2}\right) \\ -\dfrac{\alpha^2}{2(\beta+1)^2}\dfrac{(ft_c)^{2\beta+2}}{P^{2\beta+2}}\left(D^{2\beta+2}(P-D)+D(P-D)^{2\beta+2}\right) \end{array}\right]$$

$$+\frac{C}{t_c}\left[\begin{array}{c} \dfrac{\alpha}{\beta+1}\dfrac{(ft_c)^{\beta+1}}{P^{\beta+1}}\left(D(P-D)^{\beta+1}+(P-D)\beta D^{\beta+1}\right) \\ +\dfrac{\alpha^2}{2\beta+1}\dfrac{(ft_c)^{2\beta+1}}{P^{2\beta+1}}\left(\dfrac{D}{2}(P-D)^{2\beta+1}-(P-D)D^{2\beta+1}\dfrac{\beta^2}{\beta+1}\right) \\ -\dfrac{\alpha^3}{(3\beta+1)}\dfrac{(ft_c)^{3\beta+1}}{P^{3\beta+1}}\left(D(P-D)^{3\beta+1}+D^{3\beta+1}(P-D)\dfrac{\beta}{(\beta+1)}\right) \end{array}\right]$$

$$+\lambda\rho E\left[D\left(\frac{P-D}{P}\right)ft_c\right]^{-\mu+1}+\delta\rho t_c ED\left(\frac{P-D}{P}\right)\left(1-\frac{\gamma f}{\gamma+1}\right)$$
$$+\left(\frac{W}{P}+K\right)DE+WE\left(1-\frac{D}{P}\right) \tag{68}$$

Subject to the operational constraints :

$$P_{min} < P < P_{max},\quad 0 < f < 1$$

4.5 Resolution approach

The optimization structure of the extended EPQ model with variable deterioration remains consistent with that of the constant deterioration model. The objective is to minimize the average total cost function (ATC) by determining the optimal values of:

- The production rate P
- The cycle time t_c
- The fraction of the cycle with positive inventory f, where $f \in (0; 1)$

Formally, the optimization problem is defined as:

$$Minimize\ ATC(t_c, P, f)$$

s.t.

$$P_{min} \le P \le P_{max}$$
$$0 < f < 1$$

While the formulation remains structurally similar to that in Chapter 3, the use of the Weibull deterioration function with $\beta \neq 1$, introduces higher complexity and nonlinearity into the model. The time-dependent nature of deterioration affects both the inventory level expressions and their associated cost components, making closed-form analysis even more challenging.

To efficiently solve this nonlinear and potentially multi-modal problem, the Differential Evolution (DE) algorithm is again employed. Implemented via the `differential_evolution` function from the SciPy optimization module in Python, DE is well-suited for this context due to:

- The nonlinear interactions among cost components,
- The presence of exponential and power terms from the Weibull-based deterioration function,
- The existence of multiple local minima resulting from trade-offs between production, storage, and energy-related costs.

The DE algorithm operates by iteratively evolving a population of candidate solutions $X_i = (t_c, f, P)$, applying mutation, crossover, and selection operators to explore the feasible space. Infeasible candidates (e.g., those violating $f \notin (0,1)$, $P < D$) are penalized by assigning a large cost value.

As in the previous chapter, the optimization procedure is illustrated in Figure 3.2, which outlines the steps from parameter initialization to global convergence. The stopping criteria are based on either the maximum number of iterations or the absence of further improvements in ATC. This global search method ensures robust performance and is especially effective for the nonlinear cost landscape introduced by the variable deterioration rate in the extended EPQ model.

4.6 Numerical analysis

A numerical analysis has been carried out to demonstrate the model's properties. The process includes:

- Implementing the resolution procedure for a specific case using existing research data.
- Analyzing the impact of energy components on the production inventory model by comparing the energy – based EPQ model with deterioration items to the traditional EPQ model with deterioration items.
- Conducting a sensitivity analysis to examine how input parameters influence the total cost and decision variables.

4.6.1 Numerical example

The input parameters related to the manufacturing process $(i.e. D, S, H, P_{min}, P_{max}, W, K, E)$ and the input parameters related to warehousing $(i.e. \lambda, \mu, \delta, \gamma)$ have been obtained from Nguyen et al.[34] and the parameter of deterioration (α, β, C) from Pal et al. [41]. Table 4.1 summarizes the data set used in all the examples presented in this section.

Table 4.1 Input parameters for the numerical example

D	100	$unit/h$	α	0.001	
S	300	$\$/h$	β	1.2	
H	$1 \cdot 10^{-5}$	$\$/(unit \cdot h)$	λ	$445 \cdot 10^{-4}$	
W	100	kW	μ	0.23	
K	0.05	$kWh/unit$	δ	$171.2 \cdot 10^{-4}$	
E	0.2	$\$/(kWh)$	γ	0.5	
B	0.01	$\$/(unit \cdot h)$	T_w	-18	$°C$
C	3	$\$/(unit \cdot h)$	T_r	6	$°C$
P	$[150: 500]$	$unit/h$	T_{hot}	16	$°C$

Based on the given input parameters, the model yields an optimal solution with $t_c = 23.877h$, $f = 0.512$ and $P = 150\ unit/h$, resulting in a minimum average total cost of 46.815$, as illustrated in Figure 4.3.

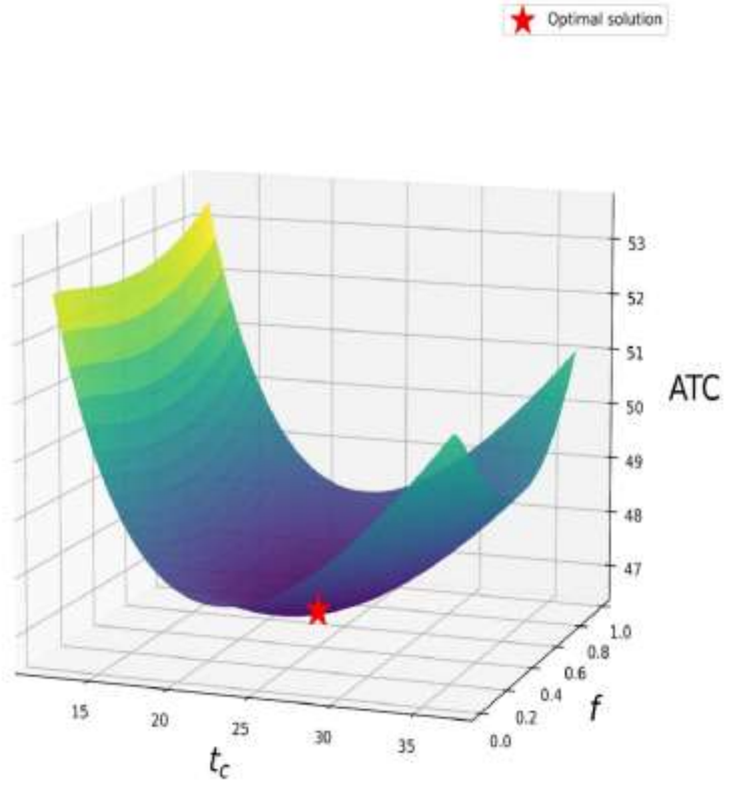

Figure 4.1 Total cost and optimal solution of fraction of period length and cycle time

Compared to the constant deterioration model presented in **Chapter 3**, the cost function in this case exhibits a more concave and complex structure. This increased complexity stems from the use of a Weibull deterioration function with $\beta \neq 1$, which introduces exponential terms into several cost components. As a result, the cost function becomes more nonlinear and prone to multiple local optimal.

A notable observation is that the model tends to select a production rate P that is close to the demand rate D. This behavior is due to the frequent appearance of P in exponential forms, which makes higher production rates significantly more expensive. Consequently, the model favors longer cycle times and extended production phases compared to the constant deterioration case.

Finally, this optimized result will serve as the basis for comparison with the traditional EPQ model, in order to evaluate whether incorporating energy consumption and time-dependent deterioration can lead to more cost-effective decisions in managing perishable inventory systems.

4.6.2 Effects of energy related factors on the classical deteriorating EPQ model

To further examine the influence of energy-related factors on production-inventory decisions, we extend the comparative framework introduced in Chapter 3 by incorporating time-dependent deterioration into both models under analysis. Specifically, we compare:

(1) The traditional EPQ model with variable deterioration **(EPQ-V)**, and

(2) The extended EPQ model with variable deterioration and energy considerations **(EEPQ-V)**.

This comparative analysis adopts the same methodology used in the previous chapter but now addresses a more realistic and complex deterioration behavior modeled by the Weibull distribution with $\beta \neq 1$. The goal is to evaluate whether integrating energy consumption in both production and storage phases under nonlinear deterioration dynamics leads to improved cost performance and more sustainable inventory strategies. In the **EPQ-V** model, the average total cost function is denoted as $ATC_{NE}(t_c, P, f)$ with cost traditional components includes Setup cost, Backorder cost, Holding cost, Deterioration cost that defined by equations (31), (34), (66) and (67). Specifically, the objective function is expressed as equation (69):

$$ATC_{NE}(t_c, f, P) = \frac{S}{t_c} + \frac{B(1-f)^2 t_c}{2} D\left(1 - \frac{D}{P}\right) \tag{69}$$

$$+\frac{H\rho}{t_c}\left[\begin{array}{c} \frac{f^2 t_c^2}{2}\frac{(P-D)D}{P} \\ +\frac{\alpha\beta}{(\beta+1)(\beta+2)}\frac{(ft_c)^{\beta+2}}{P^{\beta+2}}\left(D(P-D)^{\beta+2} - (P-D)D^{\beta+2}\right) \\ -\frac{\alpha^2}{2(\beta+1)^2}\frac{(ft_c)^{2\beta+2}}{P^{2\beta+2}}\left(D^{2\beta+2}(P-D) + D(P-D)^{2\beta+2}\right) \end{array}\right]$$

$$+\frac{C}{t_c}\left[\begin{array}{c} \frac{\alpha}{\beta+1}\frac{(ft_c)^{\beta+1}}{P^{\beta+1}}\left(D(P-D)^{\beta+1} + (P-D)\beta D^{\beta+1}\right) \\ +\frac{\alpha^2}{2\beta+1}\frac{(ft_c)^{2\beta+1}}{P^{2\beta+1}}\left(\frac{D}{2}(P-D)^{2\beta+1} - (P-D)D^{2\beta+1}\frac{\beta^2}{\beta+1}\right) \\ -\frac{\alpha^3}{(3\beta+1)}\frac{(ft_c)^{3\beta+1}}{P^{3\beta+1}}\left(D(P-D)^{3\beta+1} + D^{3\beta+1}(P-D)\frac{\beta}{(\beta+1)}\right) \end{array}\right]$$

The comparison results are presented in Table 4.2, with all input parameters specified in Table 2. The sum of setup cost and holding cost is labeled as Traditional cost ($C_{trad.}$), whereas the total energy-related costs in production and storage are denoted as Energy cost (EC) and Deterioration cost (DC).

Table 4.2 Optimal solutions and the average total cost components of two model comparison

	P^*	f^*	t_c^*	ATC	$C_{trad.}$	EC	DC
EEPQ-V	150	0.512	23.878	46.815	12.603	32.794	0.471
EPQ-V	150	0.613	66.539	60.733	4.662	52.040	2.371

While both models adopt the same optimal production rate $P^* = 150$, notable differences emerge in the cycle time and the fraction of time with positive inventory. Specifically, **EEPQ-V** results in a significantly shorter production cycle ($t_c^* = 23.878$ compared to 66.539) and a lower inventory fraction $f^* = 0.521$ versus 0.613. These findings suggest that the energy-aware model strategically shortens the cycle and reduces inventory levels to minimize energy consumption and deterioration during storage.

Table 4.3 Cost Efficiency of the Energy-Integrated Model for Deteriorating Products

	ATC	EC	DC
EEPQ - V	↓22.92%	↓36.98%	↓80.14%

In terms of overall economic performance, the **EEPQ-V** model achieves a lower average total cost of 46.815\$, representing a 22.92% reduction compared to 60.733\$ under the **EPQ-V** model. Although the traditional cost C_{trad} in the **EPQ-V** is higher in the **EEPQ-V** model (12.603\$ versus 4.662\$), this is more than offset by substantial reductions in:

- Energy cost (EC): from 52.040\$ to 32.794\$, a 36.98% decrease.
- Deterioration cost (DC): from 2.371\$ to 0.471\$, corresponding to a significant 80.14% reduction.

These results clearly demonstrate that incorporating energy consumption and time-dependent deterioration into inventory decision-making enables more efficient policies. The **EEPQ-V** model achieves this by carefully balancing production scheduling, inventory control, and environmental considerations.

In conclusion, the extended model provides a more realistic and economically superior framework for managing perishable inventory systems. It not only enhances cost efficiency but also aligns with modern goals of sustainable and energy-conscious operations, offering valuable insights for practitioners and policymakers in supply chain and production planning.

4.6.3 Sensitivity analysis

In production-inventory decision-making, uncertainties in parameter values can significantly affect overall system performance. To assess the robustness of the proposed model under such variations, sensitivity analysis is employed as a systematic tool that supports informed and resilient decision-making. This study conducts a one-at-a-time sensitivity analysis using the numerical baseline established in the previous section. In this approach, individual parameters are varied while all others are held constant, allowing for the isolated evaluation of each parameter's influence on system behavior and total cost. To facilitate interpretation, the parameters are grouped into three distinct categories:

Group 1: General production parameters (S, H, C, B, D)

Group 2: Energy consumption and deterioration factors (K, W, E, α, β)

Group 3: Inventory storage energy parameters $(T_w, \rho, \mu, \lambda, \delta, \gamma)$

To maintain consistency with the methodology used in Chapter 3, the sensitivity analysis in this section also explores changes in input parameters within a range of –25% to +25%, using a 5% increment. This approach ensures comparability across models while continuing to reflect the magnitude of fluctuations typically observed in real-world market conditions.

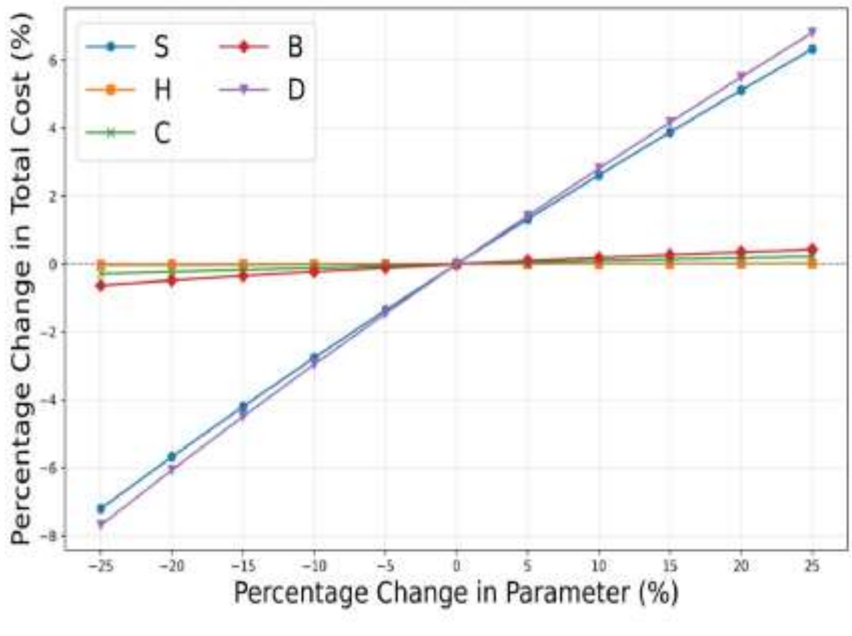

Figure 4.2 Impact of Group 1 Parameter Changes on Total Cost in Model 2

The sensitivity analysis across the three parameter groups yields several noteworthy findings. For Group 1 (general production parameters), as shown in Table 4.4 demand (D) emerges as the most influential factor affecting the total cost, followed by setup cost (S). Other parameters in this group, such as holding cost (H), unit production cost (C), and backorder cost (B), show only minor or negligible impacts. Interestingly, this influence pattern remains largely consistent with the case of constant deterioration, suggesting that market demand volatility continues to be a dominant driver of total production-inventory cost. This observation aligns with real-world industrial experience, where demand uncertainty significantly affects production planning. In parallel, setup cost also proves to be a critical factor due to its direct link with production cycle frequency and batch size.

These findings confirm that both models (constant and variable deterioration) tend to optimize decision variables by balancing cycle time and production rate, aiming to mitigate the impact of cost fluctuations and achieve greater overall cost efficiency under varying economic conditions.

Table 4.4 Results of Sensitivity Analysis for Model 2 – Group 1 Parameter Changes

Parameter	Value	P^*	t_c^*	f^*	ATC	EC	DC
S	225	150	20.611	0.501	43.443	31.262	0.376
	240	150	21.303	0.503	44.158	31.588	0.395
	255	150	21.974	0.506	44.852	31.903	0.415
	270	150	22.626	0.508	45.524	32.208	0.434
	285	150	23.260	0.510	46.178	32.505	0.452
	315	150	24.481	0.514	47.435	33.075	0.489
	330	150	25.069	0.516	48.040	33.349	0.506
	345	150	25.645	0.517	48.632	33.616	0.524
	360	150	26.209	0.519	49.210	33.878	0.541
	375	150	27.726	0.536	49.801	34.609	0.622
H	0.000075	150	23.890	0.516	46.805	32.805	0.478
	0.00008	150	23.887	0.515	46.807	32.803	0.477
	0.000085	150	23.885	0.514	46.809	32.801	0.475
	0.00009	150	23.882	0.513	46.811	32.798	0.474
	0.000095	150	23.880	0.513	46.813	32.796	0.472
	0.000105	150	23.875	0.511	46.817	32.791	0.469
	0.00011	150	23.873	0.511	46.818	32.789	0.468
	0.000115	150	23.870	0.510	46.820	32.787	0.466
	0.00012	150	23.868	0.509	46.822	32.784	0.465
	0.000125	150	23.866	0.509	46.824	32.782	0.463
C	2.25	150	24.073	0.565	46.683	32.972	0.442
	2.4	150	24.030	0.553	46.712	32.933	0.450

	2.55	150	23.989	0.542	46.739	32.896	0.457
	2.7	150	23.950	0.532	46.765	32.861	0.462
	2.85	150	23.913	0.522	46.791	32.827	0.467
	3.15	150	23.844	0.503	46.838	32.762	0.474
	3.3	150	23.812	0.494	46.860	32.731	0.476
	3.45	150	23.781	0.485	46.881	32.702	0.478
	3.6	150	23.751	0.477	46.901	32.673	0.479
	3.75	150	23.723	0.468	46.921	32.645	0.480
B	0.0075	150	24.065	0.394	46.521	32.660	0.268
	0.008	150	24.023	0.424	46.591	32.699	0.313
	0.0085	150	23.984	0.449	46.654	32.730	0.355
	0.009	150	23.946	0.472	46.712	32.755	0.396
	0.0095	150	23.911	0.493	46.765	32.776	0.434
	0.0105	150	23.846	0.529	46.860	32.809	0.506
	0.011	150	23.817	0.545	46.903	32.821	0.539
	0.0115	150	23.789	0.560	46.942	32.833	0.571
	0.012	150	23.762	0.574	46.979	32.842	0.601
	0.0125	150	23.737	0.587	47.014	32.851	0.631
D	75	150	27.439	0.490	43.214	30.980	0.379
	80	150	26.590	0.495	43.976	31.360	0.398
	85	150	25.829	0.500	44.715	31.734	0.417
	90	150	25.125	0.504	45.433	32.096	0.435
	95	150	24.477	0.508	46.133	32.449	0.453
	105	158	23.321	0.516	47.480	33.131	0.488
	110	165	22.803	0.519	48.131	33.461	0.505
	115	173	22.318	0.522	48.768	33.785	0.522
	120	180	21.864	0.525	49.391	34.102	0.538
	125	188	21.437	0.528	50.002	34.414	0.554

For Group 2 (energy consumption and deterioration factors), the sensitivity analysis once again confirms that energy is a critical cost driver in the model, consistent with the findings from the constant deterioration case. Specifically, when the unit energy cost (E) increases by 25%, the average total cost rises by more than 15%, highlighting the model's sensitivity to fluctuations in energy prices. Similarly, idle energy consumption (W) – representing the energy used during non-production phases when machines remain powered on – has the second-highest impact, with a 25% increase in

W leading to a 10% increase in total cost. These results, alongside the influence of setup cost (S) in Group 1, indicate that investing in energy-efficient machinery is an economically sound strategy for reducing overall system costs. The strong impact of Group 2 parameters is explicitly demonstrated in the results discussed following Table 4.5.

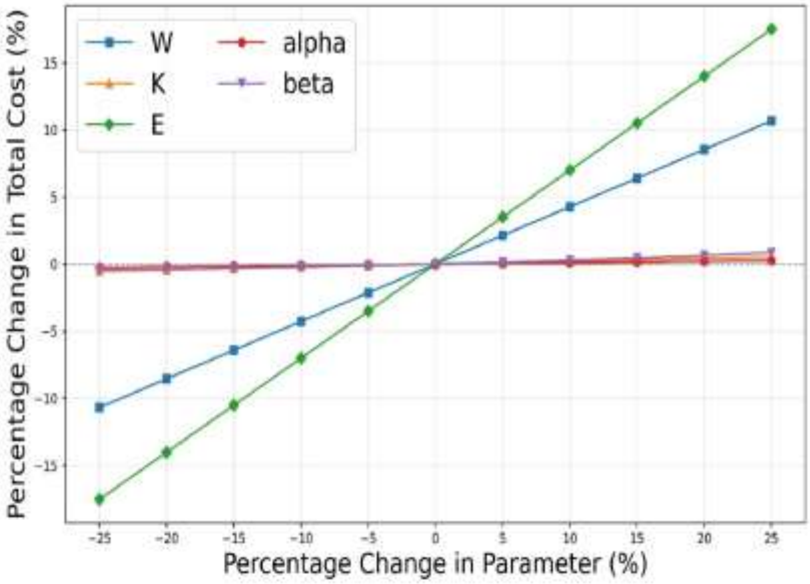

Figure 4.3 Impact of Group 2 Parameter Changes on Total Cost in Model 2

In contrast, the parameters associated with deterioration behavior over time, specifically α, β, and the unit energy consumption per product (K) have only a marginal impact on the total cost, even when increased by 25%. Although the model incorporates time-dependent deterioration through the Weibull distribution, the optimization process effectively shortens production and storage durations, thereby minimizing holding and deterioration costs. This result demonstrates the model's ability to absorb deterioration risk through strategic control of the production cycle and inventory levels, maintaining cost efficiency even in more aggressive deterioration scenarios.

Table 4.5 Results of Sensitivity Analysis for Model 2 – Group 2 Parameter Changes

Parameter	Value	P^*	t_c^*	f^*	ATC	EC	DC
	75	150	23.878	0.512	41.815	27.794	0.471
	80	150	23.878	0.512	42.815	28.794	0.471
$\boldsymbol{W}$	85	150	23.878	0.512	43.815	29.794	0.471
	90	150	23.878	0.512	44.815	30.794	0.471
	95	150	23.878	0.512	45.815	31.794	0.471

	100	150	23.878	0.512	46.815	32.794	0.471
	105	150	23.878	0.512	47.815	33.794	0.471
	110	150	23.878	0.512	48.815	34.794	0.471
	115	150	23.878	0.512	49.815	35.794	0.471
	120	150	23.878	0.512	50.815	36.794	0.471
	125	150	23.878	0.512	51.815	37.794	0.471
	0.0375	150	23.878	0.512	46.565	32.544	0.471
	0.04	150	23.877	0.512	46.615	32.594	0.471
	0.0425	150	23.878	0.512	46.665	32.644	0.471
	0.045	150	23.878	0.512	46.715	32.694	0.471
	0.0475	150	23.878	0.512	46.765	32.744	0.471
K	0.05	150	23.878	0.512	46.815	32.794	0.471
	0.0525	150	23.878	0.512	46.865	32.844	0.471
	0.055	150	23.878	0.512	46.915	32.894	0.471
	0.0575	150	23.857	0.512	46.965	32.934	0.470
	0.06	150	23.877	0.512	47.015	32.994	0.471
	0.0625	150	23.878	0.512	47.065	33.044	0.471
	0.15	150	27.120	0.554	38.431	25.768	0.653
	0.16	150	26.371	0.547	40.140	27.200	0.612
	0.17	150	25.678	0.539	41.832	28.618	0.574
	0.18	150	25.035	0.530	43.508	30.023	0.538
	0.19	150	24.437	0.521	45.168	31.414	0.504
E	0.2	150	23.878	0.512	46.815	32.794	0.471
	0.21	150	23.354	0.502	48.448	34.161	0.439
	0.22	150	22.861	0.492	50.068	35.518	0.409
	0.23	150	22.397	0.481	51.677	36.864	0.379
	0.24	150	21.958	0.469	53.274	38.198	0.351
	0.25	150	21.547	0.456	54.860	39.525	0.323
	0.00075	150	24.072	0.565	46.683	32.971	0.442
	0.0008	150	24.029	0.553	46.712	32.932	0.450
	0.00085	150	23.988	0.542	46.739	32.896	0.457
α	0.0009	150	23.950	0.532	46.766	32.860	0.462
	0.00095	150	23.913	0.522	46.791	32.826	0.467
	0.001	150	23.878	0.512	46.815	32.794	0.471
	0.00105	150	23.844	0.503	46.838	32.762	0.474

	0.0011	150	23.812	0.494	46.860	32.732	0.476
	0.00115	150	23.782	0.485	46.881	32.702	0.478
	0.0012	150	23.752	0.477	46.901	32.674	0.479
	0.00125	150	23.724	0.469	46.921	32.646	0.480
	0.9	150	24.467	0.651	46.519	33.285	0.412
	0.96	150	24.355	0.625	46.576	33.195	0.435
	1.02	150	24.237	0.598	46.636	33.100	0.452
	1.08	150	24.117	0.570	46.696	33.000	0.465
	1.14	150	23.996	0.541	46.756	32.897	0.471
β	1.2	150	23.878	0.512	46.815	32.794	0.471
	1.26	150	23.764	0.484	46.872	32.691	0.465
	1.32	150	23.657	0.456	46.926	32.592	0.455
	1.38	150	23.556	0.429	46.978	32.496	0.441
	1.44	150	23.465	0.404	47.027	32.405	0.425
	1.5	150	23.381	0.381	47.072	32.319	0.406

For Group 3 (inventory storage energy parameters), an interesting observation is that all parameters in this group exhibit a noticeable impact on the model's performance, unlike some of the low-impact variables in the previous groups (see Figure 4.4). Among them, the most influential factor is the temperature differential coefficient ρ, which captures the energy efficiency related to the warehouse's internal temperature control.

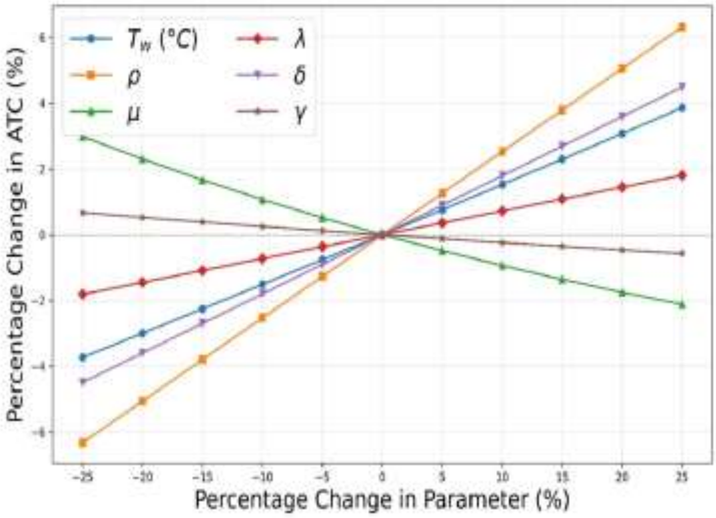

Figure 4.4 Impact of Group 3 Parameter Changes on Total Cost in Model 2

The second most impactful factors are the positive coefficients related to warehouse filling level such as δ and γ which affect energy consumption based on the amount of inventory stored. When these parameters are increased by 25%, the average total cost rises by approximately 4% to 6%, highlighting the importance of ambient temperature regulation and inventory level control in minimizing energy-related costs. To clearly demonstrate the effects of parameter variations in Group 3, Table 3.4 presents the corresponding sensitivity analysis results.

These findings emphasize that optimizing storage conditions, particularly by maintaining suitable temperatures and managing warehouse utilization effectively, can contribute meaningfully to overall cost savings in perishable inventory systems.

Table 4.6 Results of Sensitivity Analysis for Model 2 – Group 3 Parameter Changes

Parameter	Value	P^*	t_c^*	f^*	ATC	EC	DC
	-13.5	150	25.652	0.541	45.008	31.794	0.578
	-14.4	150	25.272	0.535	45.375	32.000	0.555
	-15.3	150	24.906	0.530	45.739	32.203	0.533
	-16.2	150	24.551	0.524	46.100	32.402	0.512
	-17.1	150	24.209	0.518	46.458	32.599	0.491
$\boldsymbol{T_w}$	-18	150	23.878	0.512	46.815	32.794	0.471
	-18.9	150	23.557	0.506	47.168	32.985	0.451
	-19.8	150	23.246	0.499	47.520	33.173	0.431
	-20.7	150	22.944	0.492	47.869	33.359	0.412
	-21.6	150	22.651	0.485	48.216	33.542	0.392
	-22.5	150	22.367	0.478	48.561	33.721	0.374
	2.790	150	27.138	0.558	43.669	31.029	0.662
	2.976	150	26.384	0.550	44.331	31.409	0.620
	3.162	150	25.687	0.541	44.975	31.775	0.580
	3.348	150	25.041	0.532	45.603	32.127	0.542
	3.534	150	24.439	0.522	46.216	32.466	0.505
$\boldsymbol{\rho}$	3.720	150	23.878	0.512	46.815	32.794	0.471
	3.906	150	23.351	0.501	47.400	33.109	0.438
	4.092	150	22.857	0.490	47.972	33.413	0.406
	4.278	150	22.391	0.478	48.532	33.706	0.375
	4.464	150	21.951	0.466	49.080	33.988	0.346
	4.650	150	21.535	0.453	49.617	34.259	0.317
	0.173	150	22.517	0.001	47.651	30.583	0.000
$\boldsymbol{\mu}$	0.184	150	22.908	0.286	47.661	32.481	0.124
	0.196	150	23.147	0.359	47.477	32.706	0.208

	0.207	150	23.389	0.418	47.269	32.804	0.294
	0.219	150	23.633	0.468	47.046	32.825	0.382
	0.230	150	23.878	0.512	46.815	32.794	0.471
	0.242	150	24.119	0.550	46.579	32.725	0.558
	0.253	150	24.356	0.584	46.344	32.630	0.645
	0.265	150	24.587	0.615	46.112	32.516	0.728
	0.276	150	24.810	0.642	45.884	32.590	0.809
	0.288	150	25.024	0.666	45.663	32.654	0.887
	0.0334	150	24.656	0.623	45.885	32.322	0.753
	0.0356	150	24.491	0.602	46.084	32.441	0.693
	0.0378	150	24.331	0.581	46.276	32.548	0.635
	0.0401	150	24.175	0.559	46.462	32.643	0.579
	0.0423	150	24.024	0.536	46.642	32.725	0.524
λ	0.0445	150	23.878	0.512	46.815	32.794	0.471
	0.0467	150	23.735	0.487	46.980	32.847	0.419
	0.0490	150	23.596	0.461	47.139	32.883	0.369
	0.0512	150	23.460	0.434	47.290	32.901	0.320
	0.0534	150	23.327	0.404	47.433	32.896	0.272
	0.05563	150	23.196	0.372	47.567	32.864	0.225
	0.01284	150	26.184	0.438	44.584	31.345	0.374
	0.01370	150	25.663	0.453	45.052	31.658	0.393
	0.01455	150	25.176	0.468	45.509	31.959	0.412
	0.01541	150	24.717	0.483	45.954	32.248	0.431
	0.01626	150	24.285	0.497	46.389	32.526	0.451
δ	0.01712	150	23.878	0.512	46.815	32.794	0.471
	0.01798	150	23.492	0.527	47.230	33.051	0.491
	0.01883	150	23.126	0.541	47.637	33.300	0.512
	0.01969	150	22.779	0.556	48.035	33.540	0.533
	0.02054	150	22.449	0.570	48.425	33.772	0.554
	0.02140	150	22.135	0.585	48.807	33.996	0.575
	0.375	150	23.550	0.452	47.109	32.810	0.352
	0.4	150	23.616	0.465	47.049	32.813	0.376
γ	0.425	150	23.683	0.478	46.990	32.812	0.400
	0.45	150	23.748	0.490	46.931	32.808	0.424
	0.475	150	23.813	0.501	46.873	32.802	0.447

0.5	150	23.878	0.512	46.815	32.794	0.471
0.525	150	23.941	0.523	46.757	32.783	0.494
0.55	150	24.004	0.533	46.700	32.770	0.517
0.575	150	24.067	0.542	46.644	32.756	0.539
0.6	150	24.128	0.552	46.589	32.740	0.562
0.625	150	24.189	0.561	46.534	32.723	0.584

4.7 Conclusion

Chapter 4 extended the EPQ framework by incorporating time-varying deterioration, modeled through a Weibull distribution with $\beta \neq 1$, to better reflect real-world perishability characteristics such as rapid initial spoilage or gradual degradation. While maintaining the five foundational cost components, the model exhibits higher nonlinearity due to exponential terms affecting all decision variables. The DE algorithm once again demonstrated robustness in solving the complex nonlinear problem, yielding an optimal cycle time of around 24 hours, a positive inventory fraction of approximately 50%, and a production rate close to demand, effectively minimizing storage and spoilage costs.

Comparative results showed that the extended EPQ model with energy considerations (EEPQ-V) outperforms its traditional counterpart (EPQ-D) by reducing average total cost by 22.92%, energy cost by 36.98%, and deterioration cost by 80%. Sensitivity analysis reinforced the dominant influence of energy-related parameters, especially the energy cost rate (E) and idle energy consumption (W). Additionally, warehouse-specific factors such as temperature differential coefficient (ρ) and filling-level-dependent coefficients (δ, γ) were shown to significantly affect cost, while the deterioration shape parameters (α, β) and energy load constant (K) had minimal impact due to optimal cycle adjustment.

Overall, the findings highlight the importance of jointly considering nonlinear spoilage behavior and energy efficiency in production-inventory planning. This reinforces the need for managers to adopt integrated strategies that include selecting energy-efficient machinery, optimizing cold storage environments, and fine-tuning production cycles to achieve cost-effective and sustainable operations.

CHAPTER 5 CASE STUDY AND MANAGERIAL IMPLICATIONS

In this chapter, we present the practical input parameters used for simulating both the constant and variable deterioration EPQ models. These parameters are selected based on realistic production and warehousing conditions. The Differential Evolution algorithm, as introduced in earlier sections, is employed to obtain the optimal solutions for both models under the same set of input conditions. By comparing the results derived from the two models using identical parameters, this chapter highlights the impact of incorporating variable deterioration behavior into the inventory decision-making process. The comparative analysis not only quantifies the differences in cost components and operational decisions but also provides valuable managerial insights into when and why a more complex deterioration model should be adopted in practice.

5.1 Introduction

The growing emphasis on sustainability and energy efficiency in supply chain operations has brought renewed attention to the management of perishable inventory systems. These systems often involve products with limited shelf lives and require strict environmental control, especially in cold chains, leading to substantial energy consumption and spoilage-related losses. While theoretical inventory models offer valuable analytical frameworks, their real-world applicability remains contingent upon validation through empirical or case-based studies. This chapter presents a case study aimed at evaluating the practical performance of two Economic Production Quantity (EPQ) models developed in earlier sections. The first model assumes a constant deterioration rate, consistent with traditional EPQ approaches, whereas the second introduces a variable, time-dependent deterioration rate characterized by a Weibull distribution. Both models also incorporate energy usage during production and storage phases, reflecting operational realities in modern cold supply chains.

The case study focuses on a frozen pork production and storage system located in Da Nang, Vietnam. The facility features a single-machine production process and a cold storage warehouse operating at a constant sub-zero temperature. This setting represents a realistic scenario with high perishability risks and substantial energy costs, making it an ideal testbed for evaluating the trade-offs and effectiveness of the proposed inventory models. By applying the models to this case and solving the corresponding optimization problems using a Differential Evolution (DE) algorithm, the study seeks to assess their performance in minimizing total operational costs, including holding costs, backorder penalties, spoilage losses, and energy expenses. The insights derived from this case contribute to bridging the gap between theoretical modeling and practical decision-making in energy-constrained perishable inventory systems.

5.2 Case Description

To evaluate the practical applicability of the proposed inventory models, this study considers a real-world case involving a frozen pork production facility located in Da Nang, Vietnam. The system is designed to manage perishable goods with a limited shelf life and comprises two primary stages: production and cold storage. The production stage utilizes a single cutting machine that processes pork into smaller portions. Upon slicing, the products are immediately transferred to a cold storage facility maintained at a constant temperature of -18℃ to preserve product quality [42]. Despite the controlled environment, the perishable nature of frozen pork causes deterioration to commence immediately after processing and to accelerate over time, as shown in Figure. This characteristic necessitates a responsive and cost-efficient inventory management strategy.

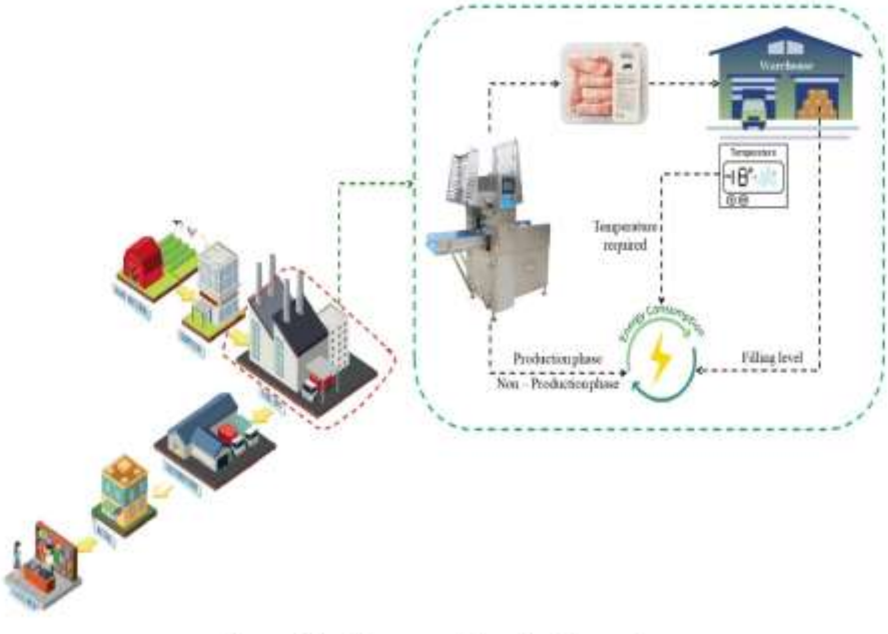

Figure 5.1 A frozen pork production system

The facility aims to minimize total operational costs, which encompass setup costs, holding costs, deterioration-related losses, backorder penalties, and energy consumption costs. Given the significant role of energy in both production and storage stages, this case study also integrates energy-related parameters into the inventory model formulation.

Empirical data were collected and calibrated from various sources to reflect realistic operational and environmental conditions. The production capacity of the cutting machine ranges from 2,100 to 5,100 slices per hour [43]. The deterioration behavior follows a Weibull distribution, with parameters adapted from Pal et al [41], Misra [25] and adjusted to reflect the specific characteristics of frozen pork. Parameters related to cold storage operations, including preservation temperature and energy efficiency, were obtained from Nguyen et al. [34]. The cost of electricity applicable to industrial use in Da Nang was referenced from [44]. Additionally, the average ambient temperature in Da Nang, 25.6°C [45],is assumed to be the external temperature affecting the energy load of the storage facility. A summary of the key operational and environmental parameters is presented in Table 5.1.

Table 5.1 Summary of Input Parameters for the Frozen Pork Case Study

D	Average hourly demand	2000	kg/h
S	Setup cost	80	$\$/h$
H	Holding cost per kilogram of pork	1	$\$/(kg \cdot h)$
W	Idle energy consumption	100	kW
K	Energy consumption for one kilogram pork producing	0.05	kWh/kg
E	Energy cost	0.08	$\$/(kWh)$
B	Backorder cost	0.01	$\$/(kg \cdot h)$
C	Unit production cost per kilogram of pork	4	$\$/(kg \cdot h)$
P	Production capacity of the machine	$[2100: 5100]$	kg/h
α	Deterioration rate	0.02	
β	Shape parameter of the Weibull distribution	$\beta = 1$ or $\beta = 1.5$	Constant Variable
λ	Positive coefficients dependent on the characteristics of the warehouse	$445 \cdot 10^{-4}$	
μ	Positive coefficients dependent on the characteristics of the warehouse, where $\mu \in (0, 1)$	0.23	
δ	Positive coefficients dependent on the filling level of the warehouse	$171.2 \cdot 10^{-4}$	
γ	Positive coefficients dependent on the filling level of the warehouse	0.5	
T_w	Expected warehouse temperature	-18	$°C$
T_r	Referenced warehouse temperature	6	$°C$
T_{hot}	Outside warehouse temperature	25	$°C$

With the input parameters outlined in Table 5.1, the frozen pork inventory system provides a representative scenario to evaluate the performance of the proposed inventory models under realistic operational and environmental conditions. Both the EPQ model with constant deterioration rate and the extended model with time-dependent deterioration (Weibull distribution) will be applied to this case. By simulating and comparing the optimal inventory decisions and associated cost structures of these two models, the study aims to highlight the practical implications of incorporating energy efficiency and deterioration dynamics into production-inventory planning.

This dual-model evaluation not only tests model validity but also informs managerial decision-making regarding model selection based on cost-efficiency and system characteristics. And with the data in Table 5.1, the case study serves as a comprehensive testbed for evaluating the two inventory models under realistic assumptions. The Differential Evolution (DE) algorithm is applied to determine optimal solutions in both models. The comparative results provide quantitative insights into how incorporating energy efficiency and variable deterioration rates can significantly influence total cost and inventory decisions in perishable supply chains.

5.3 Comparative Results and Cost Analysis

To assess the practical effectiveness of the proposed models under realistic operational conditions, both the traditional EPQ model with constant deterioration rate **(EEPQ-D)** and the extended EPQ model with variable deterioration rate and energy integration **(EEPQ-V)** are applied to the frozen pork production case introduced in Section 5.2. The simulation uses the parameter values provided in Table 5.1, and the optimization problems are solved using the Differential Evolution (DE) algorithm described in earlier chapters.

The optimal solutions and corresponding cost components for both models are summarized in Table 5.2.

Table 5.2 Summary of Results and Cost Breakdown for Two Case Scenarios

		EEPQ-D					Constant case
P^*	f^*	t_c^*	ATC	$C_{trad.}$	BC	EC	DC
2100	0.002	10.009	31.990	7.998	4.746	19.245	3.10^{-5}
		EEPQ-V					Variable case
P^*	f^*	t_c^*	ATC	$C_{trad.}$	BC	EC	DC
2100	0.9	22.605	27.752	3.539	0.108	24.105	0

From Table 5.2, the comparative results reveal several important findings:

- *Lower average total cost (ATC):* The **EEPQ-V** model achieves a 13.25% reduction in ATC (from 31.990 \$/h to 27.752 \$/h) compared to the **EEPQ-D** model. This demonstrates the cost-effectiveness of modeling deterioration as a time-dependent process.
- *Reducing deterioration cost:* The **EEPQ-V** model completely eliminates deterioration-related losses, while **EEPQ-D** still incurs small but non-negligible spoilage cost. This outcome suggests better inventory turnover and alignment with spoilage behavior under **EEPQ-V**.
- *Energy trade-off for cost efficiency:* Although the **EEPQ-V** model incurs higher energy costs (24.105 \$/h vs. 19.245 \$/h), it compensates with a significant reduction in traditional costs and backorder costs, dropping from 7.998 \$/h to 3.539 \$/h and 4.746 \$/h to 0.108 \$/h, respectively.
- *More practical production cycle:* The **EEPQ-V** model selects a longer and smoother production cycle (t_c = 22.605 h, f = 0.9), compared to an extremely short and fragmented cycle in the **EEPQ-D** model (t_c = 10.009 h, f = 0.002), making it more suitable for real-world operations.

To further investigate the models' behavior under varying spoilage levels, the deterioration rate α is varied from 0.001 (slow deterioration) to 0.02 (rapid deterioration), while keeping all other parameters unchanged. The shape parameter β remains fixed for both models.

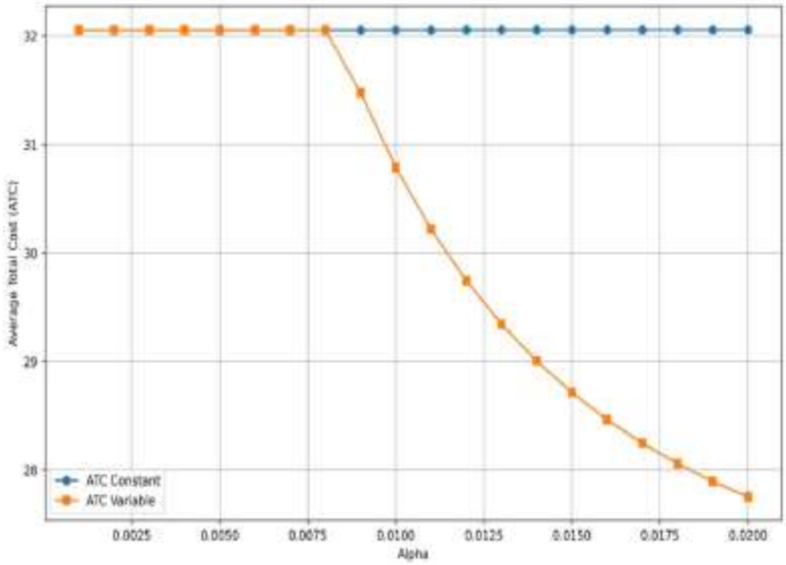

Figure 5.2 Change of deterioration rate alpha to ATC

As shown in Figure 5.1, the two models exhibit similar optimal policies when the deterioration rate is low (α < 0.008). However, as α increases, the **EEPQ-V** model adapts more effectively, maintaining a lower ATC, while the **EEPQ-D** model shows little change. This behavior aligns with the earlier sensitivity analysis indicating that the constant deterioration model is less responsive to changes in spoilage dynamics.

This result highlights that when deterioration is mild, both models may perform comparably. However, for products that degrade rapidly, the variable-rate model (**EEPQ-V**) significantly outperforms the traditional approach, both in terms of cost minimization and operational stability.

In summary, the findings confirm that incorporating time-dependent deterioration and energy cost into the inventory planning process provides a more robust and responsive strategy. The **EEPQ-V** model proves to be a superior option in managing perishable goods, especially under conditions of high spoilage risk and energy-sensitive storage requirements.

5.4 Managerial Implications and Decision Guidelines

The comparative analysis presented in Section 5.3 demonstrates that the EEPQ-V model, which incorporates a time-dependent deterioration rate and energy-related factors, offers superior performance compared to the traditional EEPQ-D model under realistic operational conditions. Notably, as the deterioration rate α increases beyond a threshold ($\alpha > 0.008$), the EEPQ-V model continues to optimize system performance by adjusting cycle length and inventory policies accordingly, whereas the EEPQ-D model fails to respond effectively due to its rigid assumption of a constant spoilage rate.

These findings underscore the critical importance of selecting an appropriate model that reflects the spoilage dynamics of the product. Furthermore, they yield several actionable insights and managerial implications for firms operating in energy-sensitive and perishable inventory environments.

The Practical Benefits of the EEPQ-V Model:

Total cost reduction through system-wide optimization: The EEPQ-V model simultaneously minimizes spoilage, backorder, and energy costs, leading to a more efficient cost structure even in scenarios of increased energy expenditure.

Improved operational stability: The longer production cycles and high inventory coverage ratio ($f \approx 0.9$) promote continuity in operations, reduce setup frequency, and lower the risk of supply interruptions.

Better alignment with the perishability profile of real-world products: The Weibull-based deterioration function captures nonlinear spoilage behavior, making the model highly suitable for time-sensitive goods such as frozen food, pharmaceuticals, and biological materials.

Support for sustainable production strategies: By integrating energy cost considerations into inventory decisions, the model facilitates the alignment of economic performance with sustainability goals, especially in cold chain and energy-intensive systems.

The managerial Guidelines and Strategic Recommendations:

Prioritize the use of EEPQ-V in systems involving fast-deteriorating or highly perishable products. The results in Figure 5.1 indicate that when the deterioration rate α exceeds 0.008, the performance gap between EEPQ-V and EEPQ-D becomes significant and economically meaningful.

Establish decision thresholds based on spoilage characteristics and storage conditions. Transitioning from the basic to the extended model should be guided by deterioration rate, energy cost levels, and production cycle length requirements to ensure quality preservation and cost efficiency.

Align investment decisions with model insights. Given the dominant share of energy in the total cost structure, firms should consider investing in high-efficiency equipment, particularly for idle phases and long-term storage operations.

In conclusion, the EEPQ-V model provides a robust, adaptable, and cost-efficient framework for managing perishable inventory systems under energy constraints. Its application not only supports short-term economic gains but also contributes to long-term sustainability objectives. The model thus serves as a valuable decision-making tool for managers seeking to balance profitability, product quality, and environmental responsibility in modern supply chains.

CHAPTER 6 CONCLUSION AND FUTURE RESEARCH DIRECTIONS

This chapter concludes the thesis by summarizing the key findings, scientific contributions, and practical implications of the proposed inventory models. It also discusses the academic recognition achieved through related publications and awards. Finally, the chapter outlines the study's limitations and suggests potential avenues for future research to further enhance the model's applicability and relevance in real-world production-inventory systems.

6.1 Conclusions and Managerial Insights

This study developed and evaluated two inventory optimization models for perishable products: the traditional Economic Production Quantity (EPQ) model with a constant deterioration rate (EEPQ-D), and an extended model that incorporates a time-dependent deterioration rate using the Weibull distribution and energy consumption parameters (EEPQ-V). These models were designed to reflect the operational realities of energy-intensive environments such as cold supply chains and were validated through a case study involving a frozen pork production system in Da Nang, Vietnam.

Comparison with conventional EPQ models reveals that integrating energy usage in both production and storage stages enables companies to achieve comprehensive optimization, significantly reducing energy costs, deterioration-related losses, and overall system costs. This highlights the critical importance of incorporating energy considerations in inventory models for perishable goods.

When both models were applied under the same parameter set, results showed that EEPQ-V outperforms EEPQ-D in terms of cost efficiency and operational stability, particularly in scenarios with nonlinear spoilage behavior and energy constraints. Specifically, the EEPQ-V model reduced the average total cost by 13.25%, completely eliminated spoilage costs, significantly lowered backorder and traditional inventory costs, and improved the feasibility of production cycles under realistic conditions. The performance gap between the two models became more pronounced with increasing deterioration rates, emphasizing the importance of selecting the appropriate model based on product characteristics.

Sensitivity analysis also indicated that demand (D), setup cost (S), energy cost (E), and warehouse temperature parameters are the most influential factors affecting system costs. Thus, firms should adopt energy-efficient equipment, optimize production schedules, and manage warehouse temperature effectively to minimize energy consumption.

6.2 Scientific and Practical Contributions

This research contributes to the literature on inventory optimization in several ways:

It integrates nonlinear deterioration behavior into the EPQ framework using the flexible Weibull distribution, enhancing realism and generalizability.

- It incorporates energy consumption from both production and warehousing stages into the cost function, offering a holistic view of system sustainability.
- It proposes a unified model capable of supporting decision-making in environments where spoilage, energy cost, and operational feasibility must be considered simultaneously.
- It demonstrates the model's applicability through a real-world case study, bridging the gap between theoretical modeling and managerial implementation.

On the practical side, the study provides a quantitative basis for choosing between simple and complex inventory models depending on input conditions such as spoilage rate and energy price. It also offers guidelines for adjusting production policies in line with environmental constraints, which is increasingly relevant under green supply chain initiatives.

6.3 Publications and award

The research outcomes of this thesis have gained academic recognition through both publication and participation in scientific conferences. Notably, a condensed version of the model developed in this thesis was presented under the title *"Economic Production Quantity Model for Deteriorating Items with Energy Considerations"* and was awarded **First Prize** at the Faculty-level Student Research Conference. The paper was officially published in the conference proceedings. Subsequently, the same work received the **Third Prize** at the University-wide Student Research Conference held at the University of Science and Technology – The University of Danang.

Furthermore, an extended and refined version of the model has been submitted to the **International Conference on Technology Innovation for Sustainable Development 2025** under the title *"Optimizing Production and Storage Decisions for Deteriorating Items in Energy-Constrained Systems."* The submission is currently under review.

These academic achievements not only validate the scientific merit of the proposed framework but also demonstrate its relevance to contemporary challenges in sustainable supply chain and production management.

6.4 Limitations and Future Research Directions

Despite the contributions of this research, certain limitations remain that pave the way for future investigations and model enhancement:

First, the present study is developed under deterministic assumptions for both demand and deterioration parameters. While this setting facilitates analytical tractability, it does not fully capture the inherent uncertainties in real-world operations. Future research should consider extending the model by incorporating stochastic demand patterns and uncertain spoilage rates through probabilistic frameworks or fuzzy logic approaches, thereby improving the robustness and practical relevance of the model.

Second, the deterioration behavior is modeled exclusively using the Weibull distribution. Although widely adopted due to its flexibility, the Weibull form may not be universally representative across all types of perishable items. Alternative statistical distributions such as the log-normal or gamma functions could be investigated to evaluate the sensitivity and applicability of the model across diverse industrial contexts.

Third, the current framework focuses on a single-item, single-machine production-inventory system. This simplification may not be suitable for more complex supply chain structures encountered in practice. Future extensions could explore multi-product, multi-stage, or networked supply chains, which would broaden the applicability of the model to integrated and large-scale systems.

In summary, while this thesis provides a foundational step toward more sustainable and responsive inventory management for perishable goods under energy constraints, there remains significant potential for further development. Subsequent research efforts can build upon this work to design more intelligent, adaptive, and environmentally-aware decision-making systems tailored for modern supply chain environments.

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